نام و نام خانوادگی ...سامان نسبتی اللهی... پاسخنامه تشریحی تکلیف شماره ۱۵... کلاس دهم ب ر A سه‌شنبه‌ها......

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$a + aq + aq^2 = 21 \Rightarrow a(1+q+q^2) = 21 \Rightarrow \frac{4}{q}(1+q+q^2) = 21 \leftarrow ①$

$a \cdot aq \cdot aq^2 = 64 \Rightarrow a^3 q^3 = 64 \Rightarrow (aq)^3 = 64 \Rightarrow aq = 4 \Rightarrow a = \frac{4}{q}$

از معادله‌ی شماره ① $\Rightarrow 4(1+q+q^2) = 21q \Rightarrow 4 + 4q + 4q^2 = 21q$

$\Rightarrow 4q^2 - 17q + 4 = 0 \to q_1 = \frac{17+15}{8} = 4 \,,\; q_2 = \frac{17-15}{8} = \frac{1}{4}$

چون q غیرصحیحی است بنابراین مقدار آن $\frac{1}{4}$ می‌باشد.

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$a_1 = x^2+4 \,,\; a_2 = 2x \,,\; a_3 = x^2-2$

$a_2^2 = a_1 a_3 \Rightarrow (2x)^2 = (x^2+4)(x^2-2) \Rightarrow x = -2$ یا 2

دو حالت برای q وجود دارد $\begin{cases} \frac{1}{2} \\ -\frac{1}{2} \end{cases}$

$r = \frac{a_2}{a_1} = \frac{2x}{x^2+4}$

چون $q > 0 \Leftarrow q = \frac{1}{2} \Leftarrow x = 2$

$a_1 = 8 \,,\; r = \frac{1}{2} \,,\; x = 2$

$S_7 = \frac{8(1-(\frac{1}{2})^7)}{1-\frac{1}{2}} = 8 \times \frac{\frac{127}{128}}{\frac{1}{2}} = \boxed{15.875}$

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۵ جمله $= a + aq + aq^2 + aq^3 + aq^4 \Rightarrow \underbrace{a}_{243}\underbrace{(1+q+q^2+q^3+q^4)}_{\frac{121}{81}}$

$\cancel{243}^{3}\left(\frac{121}{\cancel{81}_{1}}\right) = \boxed{363}$

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$a_1 = 1 \,,\; a_7 = 64 \Rightarrow d = \frac{a_m - a_n}{m-n} = \frac{63}{6} = 10.5$

$a_1 \quad a_2 \quad a_3 \quad a_4 \quad a_5 \quad a_6 \quad a_7$

$a_4 = a_1 + 3d \Rightarrow 1 + 3(10.5) = \boxed{32.5 = A}$

$a_1 = 1 \,,\; a_7 = 64 \Rightarrow a_7 = a_1 r^6 \Rightarrow \frac{a_7}{a_1} = r^6 = \frac{64}{1} = 64 \Rightarrow 2^6 \Rightarrow r = 2$

$B = a_1 r^3 = 1 \times 2^3 = \boxed{8}$

$A + B = 32.5 + 8 = \boxed{40.5}$

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$-24, -\frac{95}{4}, \ldots \Rightarrow -24, \xrightarrow{+0.25} -23.75 \qquad a_{101} = a_1 + 100d = -24 + \underbrace{100 \times 0.25}_{25} = 1$

$a_1 = 128 \,,\; a_8 = 1 \Rightarrow 128 \xrightarrow{\times \frac{1}{2}} 64, 32, 16, 8, 4, 2, 1$

$\boxed{r = \frac{1}{2}}$

۶

$a_n = a_1 + (n-1)d$

$a_3 = a_1 + 2d$

$a_7 = a_1 + 6d$

$a_9 = a_1 + 8d$

$b^2 = ac \Rightarrow (a_7)^2 = a_3 \times a_9 \Rightarrow (a_1 + 6d)^2 = (a_1 + 2d)(a_1 + 8d)$

$\Rightarrow a_1^2 + 12a_1d + 36d^2 = a_1^2 + 10a_1d + 16d^2 \Rightarrow 12a_1d + 36d^2 = 10a_1d + 16d^2$

$2a_1d + 20d^2 = 0 \Rightarrow 2d(a_1 + 10d) \Rightarrow \boxed{d = -\frac{a_1}{10}}$ البته d می‌تواند صفر باشد.

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$a_n = a + (n-1)d$

$a_2 = a + d, \quad a_4 = a + 3d, \quad a_8 = a + 7d$

$(a_4)^2 = (a_2)(a_8)$

$\Rightarrow (a + 3d)^2 = (a + d)(a + 7d) \Rightarrow a^2 + 6ad + 9d^2 = a^2 + 8ad + 7d^2$

$\Rightarrow 2d(a - d) = 0 \quad \Rightarrow a_2 = 2d, \; a_4 = 4d, \; a_8 = 8d \Rightarrow q = 2$

$t_n = \frac{1}{4} \times 2^{n-1} \Rightarrow t_{10} = \frac{1}{4} \times 2^9 = \frac{512}{4} = \boxed{128}$

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$a, ar, ar^2, ar^3, \dots$

$3ar$: سه برابر جمله‌ی دوم

$2ar^2$: دو برابر جمله‌ی سوم

ar^3 : جمله‌ی چهارم

$\Longrightarrow 2ar^2 = \frac{3ar + ar^3}{2} \Rightarrow 4ar^2 = 3ar + ar^3$

$\xrightarrow{\div ar} 4r = 3 + r^2 \Rightarrow r^2 - 4r + 3 = 0 \Rightarrow (r-1)(r-3) = 0$

r: ۱ ✗ غیرثابت غ ق ق ، $\boxed{3}$ قابل قبول

$\boxed{r = 3}$

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$2, 1/75, \dots \Rightarrow 2, \frac{7}{4}, \dots \Rightarrow d = -0/25 \Rightarrow a_n = 2 - 0/25(n-1)$

$a_4 = 1/25, \quad a_8 = 0/25, \quad a_{13} = -1$

$a_{13} + k, \; a_8 + k, \; a_4 + k \Rightarrow (a_8 + k)^2 = (a_{13} + k)(a_4 + k)$

$(0/25 + k)^2 = (-1 + k)(1/25 + k) \Rightarrow k = -5/25$

$a_{13} + k = -6/25, \quad a_8 + k = -5, \quad a_4 + k = -4$

$r = \frac{-5}{-6/25} = \boxed{0/8}$

۱۰

$t_1 = a, \quad t_4 = ar^3, \quad t_7 = ar^6 \Rightarrow a + ar^3 + ar^6 = 73$

$A_1 = a \Rightarrow A_2 = a + d = ar^3, \quad A_{10} = a + 9d = ar^6$

$A_2 = a + d = ar^3 \Rightarrow d = a(r^3 - 1) \Rightarrow d = 1(8-1) = 7$

$A_{10} = a + 9d = ar^6 \Rightarrow a + 9a(r^3 - 1) = ar^6 \Rightarrow 1 + 9(r^3 - 1) = r^6$

$r^6 = x^2, \; r^3 = x \Rightarrow 1 + 9(x-1) = x^2 \Rightarrow x^2 - 9x + 8 = 0 \Rightarrow (x-1)(x-8) = 0$

① حالت $x = 1, \; r = 1, \; r^3 = 1, \; d = 0$

② حالت $r = 2, \; r^3 = 8 \rightarrow x = 8 \rightarrow a(1 + r^3 + r^6) = 73 \Rightarrow a \cdot 73 = 73 \Rightarrow a = 1 \Rightarrow d = 1(8-1) = 7$