نام و نام خانوادگی ....علی جمشیدیان...... **پاسخنامه تشریحی تکلیف شماره** ...۱۵... **کلاس** ....پسر A...........

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$S_3=\frac{3}{2}(a_1+a_3)=21$

$\sqrt{(a_1\times a_3)^3}=64 \Rightarrow (a_1\times a_3)^3=4096 \Rightarrow a_1\times a_3=\sqrt[3]{4096}=16$

$(a_2)^3=64 \Rightarrow a_2=4$

$\Rightarrow \begin{cases} q=-2 \to -2,4,-8 \Rightarrow q=-2 \\ \cancel{q=2} \to 2,4,8 \end{cases}$

$q\notin W$

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$a_n = x^2+4,\ 2x,\ x^2-2$

$(x^2+4)(x^2-2)=4x^2 \Rightarrow x^4+2x^2-8=4x^2 \to x^4-8=2x^2 \to (x^2-2)x^2=8$

$\Rightarrow x^2=4 \Rightarrow x=\begin{cases} -2 \to 8,-4,2 \Rightarrow \cancel{q=-2}\quad q>0 \\ +2 \to 8,4,2 \Rightarrow q=2 \end{cases} \Rightarrow S_7=a_1\left(\frac{1-q^n}{1-q}\right)=8\left(\frac{1-128}{1-2}\right)=8\times127=1016$

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$1+q+q^2+q^3+q^4=\frac{121}{81}\qquad a_1=243$

$a_1+a_1q+a_1q^2+a_1q^3+a_1q^4=a_1\times\underbrace{(1+q+q^2+q^3+q^4)}_{\frac{121}{81}}=\cancel{243}^{\,3}\times\frac{121}{\cancel{81}}=363$

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$t_7=64\quad t_1=1 \Rightarrow 6d=63 \Rightarrow d=10.5\qquad t_4=A=t_1+3d=1+31.5=32.5$

$a_7=64\quad a_1=1 \Rightarrow q^6=64 \Rightarrow q=\pm 2 \Rightarrow a_4=B=t_1q^3=\begin{cases} 1\times(-8)=-8 \\ 1\times 8=+8 \end{cases}$

$\Rightarrow A+B=\begin{cases} 32.5+8=40.5 \\ 32.5-8=24.5 \end{cases}$

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$t_n: -24, -\frac{95}{4}, \dots \Rightarrow d=-\frac{95}{4}-(-24)=\frac{1}{4} \Rightarrow t_{101}=-24+100d=-24+25=1$

$a_n: 128, a_2, \dots \to a_8=128\cdot q^7=1 \Rightarrow q^7=\frac{1}{128} \Rightarrow q=\frac{1}{2}$

$t_3, t_7, t_9 = a_1, a_2, a_3$ $(t_1+6d)^2 = (t_1+2d)(t_1+8d) \to \cancel{t_1^2} + \underbrace{\cancel{12t_1d}}_{2t_1d} + \underbrace{\cancel{36d^2}}_{20d^2} = \cancel{t_1^2} + \cancel{10t_1d} + \cancel{16d^2} \Rightarrow 2t_1d + 20d^2 = 0 \xrightarrow{d\neq0} 2t_1 = -20d \Rightarrow d = -\frac{1}{10}t_1$	۶
$t_2, t_4, t_8 = a_n, a_{n+1}, a_{n+2}$ $\Rightarrow (t_1+3d)^2 = (t_1+d)(t_1+7d) \to \cancel{t_1^2} + \cancel{6t_1d} + \underbrace{\cancel{9d^2}}_{2d^2} = \cancel{t_1^2} + \underbrace{\cancel{8t_1d}}_{2t_1d} + \cancel{7d^2} \Rightarrow 2d^2 = 2t_1d$ $\xrightarrow{d\neq0} 2d = 2t_1 \Rightarrow d = t_1 \Rightarrow a_n, a_{n+1}, a_{n+2} = 2t_1, 4t_1, 8t_1 \Rightarrow q = \frac{4t_1}{2t_1} = 2$ $\xrightarrow{q=2} a_1 = \frac14, a_{10} = a_1q^9 = \frac14 \times 2^9 = 2^7 = 128$	۷
$3a_2, 2a_3, a_4 = t_1, t_2, t_3 \Rightarrow \frac{3a_1q + a_1q^3}{2} = 2a_1q^2 \Rightarrow 3a_1q + a_1q^3 = 4a_1q^2$ $\xrightarrow{q\neq0} 3a_1 + a_1q^2 = 4a_1q \to a_1(3+q^2) = 4a_1q \xrightarrow{a_1\neq0} 3+q^2 = 4q \to q^2 - 4q + 3 = 0$ $= (q-1)(q-3) = 0 \Rightarrow q = \begin{cases} 3 \\ 1 \end{cases}$	۸
$t_n = 2, \frac74, \dots \Rightarrow d = -\frac14 \Rightarrow t_4 = 2 + 3\times(-\frac14) = \frac54, \ t_8 = 2 + 7\times(-\frac14) = \frac14, \ t_{13} = 2 + 12\times(-\frac14) = -1$ $\frac54 + x, \frac14 + x, -1 + x = a_1, a_2, a_3 \Rightarrow (\frac14 + x)^2 = (\frac54 + x)(-1 + x) \to \underbrace{\cancel{\frac{1}{16}}}_{\frac{21}{16}} + \underbrace{\cancel{\frac12 x}}_{\frac14 x} + \cancel{x^2} = \cancel{x^2} + \cancel{\frac14 x} - \cancel{\frac54}$ $\to \frac14 x + \frac{21}{16} = 0 \Rightarrow x = -5{,}25 \Rightarrow q = \frac{\frac14 - \frac{21}{4}}{\frac54 - \frac{21}{4}} = \frac{-\frac{20}{4}}{-\frac{16}{4}} = \frac{20}{16} = \frac54$	۹
$a_1, a_4, a_7 = t_1, t_2, t_{10} \Rightarrow t_2^2 = t_1 \times t_{10} \to (t_1+d)^2 = t_1 \times (t_1 + 9d) \to \cancel{t_1^2} + \cancel{2t_1d} + d^2 = \cancel{t_1^2} + \underbrace{\cancel{9t_1d}}_{7t_1d}$ $\xrightarrow{d\neq0} d = 7t_1 \to t_1 + t_2 + t_{10} = 73 = t_1 + 8t_1 + 64t_1 = 73t_1 \Rightarrow t_1 = 1 \Rightarrow d = 7\times1 = 7$	۱۰