نام و نام خانوادگی : سپهر بدالی   تکالیف شماره : ۱۵   نام کلاس : دهم پسر A

۱

a, aq, aq^2

$a \times aq \times aq^2 = a^3q^3 \longrightarrow (aq)^3 = 64$

$aq = 4$

$a = \frac{4}{q}$

$a + aq + aq^2 = 21$

$a(1+q+q^2) = 21$

$\left[\frac{4}{q}(1+q+q^2) = 21\right] \times q$

$4(1+q+q^2) = 21q$

$4q^2 - 17q + 4 = 0$

$\Delta = b^2 - 4ac = 17^2 - 4(4)(4) = 225$

$225 = 15^2$

$q = \frac{17 \pm 15}{8}$

$q = 4$ ✗ $\boxed{q = \frac{1}{4}}$

۲

$a_1 = x^2+4 \quad a_2 = 2x \quad a_3 = x^2-2$

$\frac{a_2}{a_1} = \frac{a_3}{a_2}$

$\frac{2x}{x^2+4} = \frac{x^2-2}{2x}$

$4x^2 = x^4 + 2x^2 - 8$

$x^4 - 2x^2 - 8 = 0$

$x^2 - 2x - 8 =$

$x = \frac{2 \pm \sqrt{4+32}}{2}$

$x = \frac{2 \pm 6}{2}$

$x_2 = -2$ ✗

$x^2 = 4 \longrightarrow x = \pm 2$

$x = 2$: $8, 4, 2$ ✓ $x = -2$: $8, -4, 2$ ✗

$S_7 = a_1 \frac{1-q^7}{1-q}$

$S_7 = 8 \times \frac{1-(\frac{1}{2})^7}{1-\frac{1}{2}}$

$\boxed{\frac{2032}{128}}$

۳

$a_1 = 243$

$S_5 = a_1(1+q+q^2+q^3+q^4)$

$243 \times \left(\frac{121}{81}\right) = S_5$

$\frac{29403}{81} = 363$

۴

$d = \frac{64-1}{5+1} = \frac{63}{6} = 10.5$

$A = a_1 + 3d = 1 + 31.5 = 32.5$

$q^{n+1} = \frac{b}{a}$

$q^6 = \frac{64}{1} = 64$

$q = 2 \quad B = 1 \times q^3 = 2^3 = 8$

$A + B = 32.5 + 8 = 40.5$

۵

$a_1 = -24 \quad a_2 = \frac{-95}{4}$

$d = a_2 - a_1 = -\frac{95}{4} - (-24)$

$d = \frac{-95+96}{4} = \frac{1}{4}$

$a_n = a_1 + (n-1)d$

$a_{101} = a_1 + 100d$

$-24 + 25 = 1$

$t_8 = 1$

$t_8 = t_1 q^7$

$1 = 128q^7$

$q^7 = \frac{1}{128}$

$\boxed{q = \frac{1}{2}}$

6

$a_3 = a_1 + 2d \qquad a_7 = a_1 + 6d \qquad a_9 = a_1 + 8d$

$(a_7)^2 = a_3 \cdot a_9 \longrightarrow$ در تصاعد هندسی

$(a_1+6d) = (a_1+2d)(a_1+8d)$

$\cancel{a_1^2} + 12a_1d + 36d^2 = \cancel{a_1^2} + 10a_1d + 16d^2$

$2a_1d + 20d^2 = 0 \xrightarrow{\div 2} a_1d + 10d^2 = 0 \longrightarrow d = -\frac{a_1}{10}$

7

a_2 , a_4 , a_8

$a_2 = a + d$

$a_4 = a + 3d$

$a_8 = a + 7d$

$\frac{a+3d}{a+d} = q$

$\frac{a+7d}{a+3d} = q$

$\frac{a+3d}{a+d} = \frac{a+7d}{a+3d}$

$(a+3d)^2 = (a+d)(a+7d)$

$a^2 + 6ad + 9d^2 = a^2 + 8ad + 7d^2$

$-2ad + 2d^2 = 0$

$2d(d-a) = 0$

$d - a = 0$

$d = a$

$a + 3d = 4d$

$a + d = 2d$

$q = \frac{4d}{2d} = 2$

$t_{10} = \frac{1}{4} \times 2^9 = \boxed{128}$

8

$3aq , 2aq^2 , aq^3$

(زیر $3aq$:) ۲ جمله برابر۳ — (زیر $2aq^2$:) دوبرابر جمله ۳ — ($aq^3 \rightarrow$) جمله ۴

$2(2aq^2) = 3aq + aq^3$

$a \neq 0$

$4aq^2 = 3aq + aq^3$

$4q^2 = 3q + q^3 \quad \div q$

$q^3 - 4q^2 + 3q = 0$

$q(q^2 - 4q + 3) = 0$

$q = 0$ ✗ یا $q^2 - 4q + 3 = 0$

$(q-1)(q-3) = 0$

$q = 1$ ✗ (دنباله هندسی غیرثابت) $\quad \boxed{q = 3}$

9

$d = \frac{7}{4} - 2 = -\frac{1}{4}$

$a_8 = x$

$a_4 = x - 4d$

$a_{13} = x + 5d$

$(a_8 + y)^2 = (a_4 + y)(a_{13} + y)$

$(x+y)^2 = (x - 4d + y)(x + 5d + y)$

$\cancel{x^2} + \cancel{2xy} + \cancel{y^2} = \cancel{x^2} + dx + \cancel{2xy} - 20d^2 + dy + \cancel{y^2}$

$dx + dy + 20d^2 = 0 \Rightarrow d(x + y - 20d) = 0 \quad d \neq 0$

$x + y = 20d \rightarrow a_8 + y = 20d$

$q = \frac{a_8 + k}{a_{13} + k} = \frac{20d}{20d + 5d} = \frac{20}{25}$

$\frac{20}{25} = \boxed{\frac{5}{4}}$

10

$t_1 = a \qquad t_4 = aq^3 \qquad t_7 = aq^6$

$a(1 + q^3 + q^6) = 73$

$a_1 = a \qquad a_2 = aq^3 \qquad a_{10} = aq^6$

$a_{10} = a_1 + 9d$

$a_2 = a_1 + d$

$d = a_1(q^3 - 1)$

$a_{10} = a_1q^6 = a_1 + 9d = a_1 + 9(a_1(q^3-1)) = a_1(1 + 9(q^3-1))$

$\cancel{a_1}q^6 = \cancel{a_1}(1 + 9(q^3-1)) \qquad d = a_1(q^3-1) = \boxed{7}$

$q^6 = 1 + 9(q^3 - 1)$

$q^6 = 9q^3 - 8$

با فرض $q^3 = x$: $\quad x^2 - 9x + 8 = 0 \quad x = 8 , \; x = 1 \quad q = 2$

$a_1(1 + 8 + 64) = 73$

$a_1 = 1$