دهم تسیر ۱

پارسا رحالی

تکالیف شماره (۱۵)

۱)

$$a_1\left(\frac{q^3-1}{q-1}\right)=21 \Longrightarrow a(q^2+q+1)=21 \Rightarrow a_1q^2+a_1q+a_1=21 \Rightarrow 4q+4+\frac{4}{q}=21$$

$$a_1\times a_1q\times a_1q^2=64 \Rightarrow a_1^3q^3=64 \Rightarrow a_1q=4 \Rightarrow \underbrace{4q^2+4q+4=21q \Rightarrow 4q^2-17q+4=0}$$

$$q^2-17q+16=0$$

$$(q-1)(q-16) \Rightarrow q \begin{cases} \frac{1}{4}\ \times \\ \frac{16}{4}=4\ \checkmark \end{cases}$$

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۲)

$$x^2+4,\ 2x,\ x^2-2 \Rightarrow (x^2+4)(x^2-2)=(2x)^2 \Rightarrow x^4+2x^2-8=4x^2$$

$$\Rightarrow x^4-2x^2-8=0 \Rightarrow (x^2-4)(x^2+2)=0 \Rightarrow x^2 \begin{cases} -2 \quad \text{غ} \\ 4 \Rightarrow x=\pm 2 \end{cases}$$

$$a_n \begin{cases} 8,4,2 \\ 8,-4,2 \end{cases} \ \overset{q>0}{\Longrightarrow} a_n=8,4,2 \Rightarrow q=\frac{1}{2}$$

$$S_7=8\left(\frac{1-(\frac{1}{2})^7}{1-\frac{1}{2}}\right)=8\times\frac{\frac{127}{\cancel{128}_{64}}}{\frac{1}{\cancel{2}_1}}=\cancel{8}\times\frac{127}{\cancel{64}_8}=\frac{127}{8}$$

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۳)

$$1+q+q^2+q^3+q^4=\frac{121}{81} \qquad a_1=243$$

$$S_5=a_1+a_1q+a_1q^2+a_1q^3+a_1q^4=a_1(1+q+q^2+q^3+q^4)=\overset{3}{\cancel{243}}\times\frac{121}{\cancel{81}}=363$$

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۴)

$$d=\frac{64-1}{6}=10.5 \qquad a_n=1,\ 11.5,\ 22,\ \underbrace{32.5}_{A},\ 43,\ 53.5,\ 64$$

$$q^6=\frac{64}{1} \Rightarrow q=\pm 2 \Rightarrow b_n \begin{cases} 1,2,4,8,16,32,64 \\ 1,-2,4,-8,16,-32,64 \end{cases}$$

B

$$A+B \begin{cases} 32.5+8=40.5 \\ 32.5-8=24.5 \end{cases}$$

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۵)

$$-24,\ -\frac{95}{4},\dots \Rightarrow a_n=-\frac{96}{4},\ -\frac{95}{4},\dots \Rightarrow a_n=\frac{n-97}{4} \Rightarrow a_{101}=\frac{4}{4}=1$$

$$\underbrace{128,a_2}_{\div 2},\underbrace{a_2,a_3}_{\div 2},\underbrace{a_3,a_4}_{\div 2},\underbrace{a_4,a_5}_{\div 2},\underbrace{a_5,a_6}_{\div 2},\underbrace{a_6,a_7}_{\div 2},\underbrace{a_7,1}_{\div 2},\dots \Rightarrow q^7=\frac{1}{128} \Rightarrow q=\frac{1}{2}$$

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۶)

$$(a+2d),(a+6d),(a+8d) \Rightarrow a^2+10ad+16d^2=a^2+12ad+36d^2 \Rightarrow$$

$$20d^2+2ad=0 \Rightarrow 2d(10d+a)=0 \quad d \begin{cases} 0 \\ \frac{-a}{10} \end{cases}$$

۷)

$a+d\ ,\ a+3d\ ,\ a+7d \Rightarrow a^2+8ad+7d^2 = a^2+6ad+9d^2 \Rightarrow$

$2d^2-2ad=0 \Rightarrow 2d(d-a)=0 \Rightarrow d \begin{cases} 0 & \times \\ a & \checkmark \end{cases}$

$\left.\begin{aligned} a+d=2a=\frac{1}{4} &\Rightarrow a=\frac{1}{8} \\ a+3d=4\times\frac{1}{8}=\frac{1}{2} & \end{aligned}\right\} \Rightarrow q=\frac{\frac{1}{2}}{\frac{1}{4}}=2 \Rightarrow a_{10}=a_1q^9=\frac{1}{4}\times 2^9=128$

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۸)

$3a_1q\ ,\ 2a_1q^2\ ,\ a_1q^3 \Rightarrow 3a_1q+a_1q^3=2\times 2a_1q^2$

$\Rightarrow \cancel{a_1q}(3+q^2)=\cancel{4a_1q^2}_{4q} \Rightarrow 3+q^2=4q \Rightarrow q^2-4q+3=0 \Rightarrow$

$(q-3)(q-1)=0 \Rightarrow q \begin{cases} 1 & \times \\ 3 & \checkmark \end{cases}$

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۹)

$2, \frac{7}{4}, \ldots \Rightarrow a_n=\frac{9-n}{4} \Rightarrow \begin{cases} a_4=\frac{5}{4} \\ a_8=\frac{1}{4} \\ a_{13}=\frac{-4}{4} \end{cases}$

$b_n=\frac{5}{4}+x\ ,\ \frac{1}{4}+x\ ,\ \frac{-4}{4}+x \Rightarrow \cancel{x^2}+\frac{x}{4}-\frac{5}{4}=\cancel{x^2}+\frac{x}{2}+\frac{1}{16} \Rightarrow$

$4x-20=8x+1 \Rightarrow 4x=-21 \Rightarrow x=\frac{-21}{4}$

$b_n=\frac{5}{4}+x\ ,\ \frac{1}{4}+x\ ,\ \frac{-4}{4}+x \Rightarrow -4, -5, -\frac{25}{4} \Rightarrow q=\frac{-5}{-4}=1.25$

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۱۰)

$\underbrace{a_1+a_1q^3}_{\times q^3}$, $\underbrace{a_1q^3+a_1q^6}_{\times q^3}$

$a_1+a_1q^3+a_1q^6=73 \Rightarrow a_1(1+q^3+q^6)=73$

دنباله حسابی: $\underset{(1)}{a_1}\quad \underset{(2)}{a_1q^3}\ \ldots\ \underset{(10)}{a_1q^6} \Rightarrow \left.\begin{aligned} d&=a_1q^3-a_1 \\ d&=\frac{a_1q^6-a_1q^3}{8} \end{aligned}\right\} \Rightarrow \cancel{a_1}(q^3-1)=\frac{\cancel{a_1}q^3\cancel{(q^3-1)}}{8}$

$\Rightarrow \frac{q^3}{8}=1 \Rightarrow q=2$

$a_1(\overbrace{1+q^3+q^6}^{q=2})=73 \Rightarrow a_1(73)=73 \Rightarrow a_1=1$

$d=a_1q^3-a_1 \Rightarrow d=8-1=7$

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