دهم تجربی ۱۱ ۱۹٫۲۵ تکلیف شماره (۱۵) پارسا رجالی

۱) ۱٫۷۵

$a_1\left(\frac{q^3-1}{q-1}\right)=21 \Longrightarrow a(q^2+q+1)=21 \Rightarrow a_1q^2+a_1q+a_1=21 \Rightarrow 4q+4+\frac{4}{q}=21$

$a_1\times a_1q\times a_1q^2=64 \Rightarrow a_1^3q^3=64 \Rightarrow a_1q=4 \Rightarrow 4q^2+4q+4=21q \Rightarrow 4q^2-17q+4=0$

$q^2-17q+16=0$

$(q-1)(q-16) \Rightarrow q = \frac{1}{4}$ ✓ , $\frac{16}{4}=4$ ✗

q باید غیرحسابی باشه!

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۲) (۲)

$x^2+4, 2x, x^2-2 \Rightarrow (x^2+4)(x^2-2)=(2x)^2 \Rightarrow x^4+2x^2-8=4x^2$

$\Rightarrow x^4-2x^2-8=0 \Rightarrow (x^2-4)(x^2+2)=0 \Rightarrow x^2 = -2$ غ ، $4 \Rightarrow x=\pm 2$

a_n: $8, 4, 2$ / $8, -4, 2$; $q>0 \Rightarrow a_n=8,4,2 \Rightarrow q=\frac{1}{2}$

$S_7=8\left(\frac{1-(\frac{1}{2})^7}{1-\frac{1}{2}}\right)=8\times\frac{\frac{127}{128}}{\frac{1}{2}}=8\times\frac{127}{64}=\frac{127}{8}$ ✓

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۳) (۲)

$1+q+q^2+q^3+q^4=\frac{121}{81} \quad a_1=243$

$S_5=a_1+a_1q+a_1q^2+a_1q^3+a_1q^4=a_1(1+q+q^2+q^3+q^4)=243\times\frac{121}{81}=363$ ✓

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۴) (۲)

$d=\frac{64-1}{6}=10/5 \quad a_n=1, 11/5, 22, \underbrace{32/5}_{A}, 43, 53/5, 64$

$q^6=\frac{64}{1} \Rightarrow q=\pm 2 \Rightarrow b_n$: $1,2,4,8,16,32,64$ / $1,-2,4,-8,16,-32,64$; B

$A+B$: $32/5+8=40/5$ ✓ ، $32/5-8=24/5$ ✓

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۵) (۲)

$-24, -\frac{95}{4}, \dots \Rightarrow a_n=\frac{-96}{4}, \frac{-95}{4}, \dots \Rightarrow a_n=\frac{n-97}{4} \Rightarrow a_{101}=\frac{4}{4}=1$

$128, a_2, a_3, a_4, a_5, a_6, a_7, 1, \dots$ ($\div 2$) $\Rightarrow q^7=\frac{1}{128} \Rightarrow q=\frac{1}{2}$ ✓

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۶) (۲)

$(a+2d),(a+6d),(a+8d) \Rightarrow a^2+10ad+16d^2=a^2+12ad+36d^2 \Rightarrow$

$20d^2+2ad=0 \Rightarrow 2d(10d+a)=0 \quad d = 0 ,\ \frac{-a}{10}$ ✓

۷) (۲)

$$a+d\,,\ a+3d\,,\ a+7d \Rightarrow a^2+8ad+7d^2=a^2+6ad+9d^2 \Rightarrow$$

$$2d^2-2ad=0 \Rightarrow 2d(d-a)=0 \Rightarrow d \begin{cases} 0 & \times \\ a & \checkmark \end{cases}$$

$$\left.\begin{array}{l} a+d=2a=\frac{1}{4} \Rightarrow a=\frac{1}{8} \\ a+3d=4\times\frac{1}{8}=\frac{1}{2} \end{array}\right\} \Rightarrow q=\frac{\frac{1}{2}}{\frac{1}{4}}=2 \Rightarrow a_{10}=a_1q^9=\frac{1}{4}\times 2^9=128 \checkmark$$

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۸) (۲)

$$3a_1q\,,\ 2a_1q^2\,,\ a_1q^3 \Rightarrow 3a_1q+a_1q^3=2\times 2a_1q^2$$

$$\Rightarrow \cancel{a_1q}(3+q^2)=\cancel{4a_1q^2}_{\,4q} \Rightarrow 3+q^2=4q \Rightarrow q^2-4q+3=0 \Rightarrow$$

$$(q-3)(q-1)=0 \Rightarrow q \begin{cases} 1 & \times \\ 3 & \checkmark \end{cases} \checkmark$$

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۹) (۲)

$$2,\frac{7}{4},\dots \Rightarrow a_n=\frac{9-n}{4} \Rightarrow \begin{cases} a_4=\frac{5}{4} \\ a_8=\frac{1}{4} \\ a_{13}=\frac{-4}{4} \end{cases}$$

$$b_n=\frac{5}{4}+x\,,\ \frac{1}{4}+x\,,\ \frac{-4}{4}+x \Rightarrow \cancel{x^2}+\frac{x}{4}-\frac{5}{4}=\cancel{x^2}+\frac{x}{2}+\frac{1}{16} \Rightarrow$$

$$4x-20=8x+1 \Rightarrow 4x=-21 \Rightarrow x=\frac{-21}{4}$$

$$b_n=\frac{5}{4}+x\,,\ \frac{1}{4}+x\,,\ \frac{-4}{4}+x \Rightarrow -4,-5,-\frac{25}{4} \Rightarrow q=\frac{-5}{-4}=1.25 \checkmark$$

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۱۰) (۱٫۵)

$$\underbrace{a_1+a_1q^3}_{\times q^3}\!\!\underbrace{+\,a_1q^6}_{\times q^3}=73 \Rightarrow a_1(1+q^3+q^6)=73$$

دنباله حسابی: $\underset{(1)}{a_1}\quad \underset{(2)}{a_1q^3}\ \dots\ \underset{(10)}{a_1q^6}$

$$\Rightarrow \left.\begin{array}{l} d=a_1q^3-a_1 \\ d=\dfrac{a_1q^6-a_1q^3}{8} \end{array}\right\} \Rightarrow \cancel{a_1}(\cancel{q^3-1})=\frac{\cancel{a_1}q^3(\cancel{q^3-1})}{8}$$

$$\Rightarrow \frac{q^3}{8}=1 \Rightarrow q=2$$

$$a_1(\overbrace{1+q^3+q^6}^{q=2})=73 \Rightarrow a_1(73)=73 \Rightarrow a_1=1$$

$$d=a_1q^3-a_1 \Rightarrow d=8-1=7 \checkmark$$

حالت دوم (دنباله ثابت): $q=1 \rightarrow d=0$

پایان