حسابان نوآوری - دهم پسرها - تکلیف شماره ۱۵

۱-

$a \times aq \times aq^2 = 64 \Rightarrow aq = 4 \Rightarrow a \times aq^2 = 16$

$a(1+q+q^2) = 21$

$\Rightarrow \frac{1+q+q^2}{aq^2} = \frac{21}{16} \Rightarrow \frac{1+q+q^2}{4q} = \frac{21}{16} \Rightarrow 16q^2 - 68q + 16 = 0 \Rightarrow 4q^2 - 17q + 4 = 0$

$\xrightarrow{\text{روش}} q^2 - 17q + 16 = 0 \Rightarrow (q-1)(q-16) = 0 \Rightarrow q = 1, q = 16 \xrightarrow{\div 4} \boxed{q = \frac{1}{4}}, \underbrace{q = 4}_{\text{قبول}}$

۲-

$(x^2-2)(x^2+4) = 4x^2 \Rightarrow x^4 + 2x^2 - 8 = 4x^2 \Rightarrow x^4 - 2x^2 - 8 = 0 \Rightarrow (x^2-4)(x^2+2) = 0$

$\Rightarrow x = \sqrt{2}, \underbrace{-\sqrt{2}, -2}_{\text{غیرقابل}}, 2 \rightarrow 8, 4, 2, \dots$

$\underbrace{6, 2\sqrt{2}, 0}_{\text{قابل}}$

$\Rightarrow S_7 = 8\left(\frac{(\frac{1}{2})^7 - 1}{\frac{1}{2} - 1}\right) = 8 \frac{-\frac{127}{128}}{-\frac{1}{2}} = \cancel{8}\left(\frac{127}{\frac{\cancel{64}}{8}}\right) = \frac{127}{8} = 15.875$

۳-

$S_5 = a(1+q+q^2+q^3+q^4) = \cancel{243}^{3} \times \frac{121}{\cancel{81}} = \boxed{363}$

۴-

$A = \frac{64-1}{2} = 22.5$

$B = \sqrt{64+1} = \pm 8 \rightarrow \underbrace{-8}_{\text{قابل}}, 8$

$A + B = 22.5 + 8 = 30.5$

۵-

$a_n = \underbrace{-24, -24.25}_{0.25}, \dots \Rightarrow a_n = \frac{1}{4}n - 24.25 \Rightarrow a_{101} = 1$

$b_n = 128, a_2, \dots, a_8 = 1 \Rightarrow q^7 = \frac{1}{128} \Rightarrow q = \frac{1}{\sqrt[7]{2^7}} \Rightarrow \boxed{q = \frac{1}{2}}$

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۶-

$(a+2d)(a+8d)=(a+4d)^2$

$a^2+10ad+16d^2=a^2+12ad+16d^2 \rightarrow a+10d=0 \Rightarrow d=-\frac{a}{10}$

$\Rightarrow -2ad-2d^2=0 \Rightarrow -2d(a+d)=0$

$d=0 \rightarrow$ غیرممکن

۷-

$(a+d)(a+7d)=(a+3d)^2$

$\Rightarrow a^2+8ad+7d^2=a^2+6ad+9d^2$

$\Rightarrow 2ad-2d^2=0 \Rightarrow 2d(a-d)=0 \rightarrow a=d$

$d=0 \rightarrow$ غیرممکن $\Rightarrow$ ۵۵۴

$a=d$ $\quad \underbrace{2d, 4d}_{\times 2}, \underbrace{4d, 8d}_{\times 2}, \ldots \Rightarrow q=2 \Rightarrow a_{10}=b_4\times 2^9=\boxed{128}$

۸-

$\frac{2aq+aq^3}{2}=2aq^2$

$\Rightarrow aq(q^2+3)=4aq^2$

$\Rightarrow \underbrace{aq}_{\neq 0}\underbrace{(q^2-4q+3)}=0$

$\hookrightarrow q^2-4q+3=0 \Rightarrow (q-3)\overbrace{(q-1)}^{\text{غ ق ق}}=0 \Rightarrow \boxed{q=3}$

۹-

$(x+a_4)(x+a_{12})=(x+a_8)^2 \Rightarrow (x+\frac{d}{4})(x-1)=(x+\frac{1}{4})^2$

$\Rightarrow \cancel{x^2}+\frac{1}{4}x-\frac{d}{4}=\cancel{x^2}+\frac{1}{2}x+\frac{1}{16} \Rightarrow -\frac{1}{4}x=\frac{21}{16} \Rightarrow x=-\frac{21}{4}$

$a_n=\underbrace{-4, -d}_{\times \frac{d}{4}}, \underbrace{-d, -\frac{2d}{4}}_{\times \frac{d}{4}}, \ldots \Rightarrow \boxed{q=\frac{d}{4}}$

۱۰-

I) $\rightarrow q^3=\frac{10-2}{2-1}=8 \Rightarrow q=2 \Rightarrow a_2=8, a_4=32, a_6=64 \Rightarrow a=1, a_4=8, a_6=64$

$\Rightarrow a_1=1, a_4=8, a_{10}=64 \Rightarrow \boxed{d=7}$

II) $\rightarrow a(a+7d)=(a+d)^2 \Rightarrow a^2+7ad=a^2+2ad+d^2 \Rightarrow d(7a-d)=0 \begin{cases} 7a-d=0 \\ d=0 \end{cases}$

$7a-d=0 \Rightarrow a=\frac{1}{7}d \Rightarrow \frac{1}{7}d+\frac{8}{7}d+\frac{64}{7}d=7c \Rightarrow \frac{7c}{7}d=7c \Rightarrow d=7$