نام و نام خانوادگی ...نیلا نجاتی... پاسخنامه تشریحی تکلیف شماره ..۱۵.. کلاس ..۸...

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$a_1+a_2+a_3 = a_1+a_3 = 21 - a_2 = 17$

$a_1 a_2 a_3 = 64 \Rightarrow a^3 q^3 = 64 \Rightarrow aq = 4 = a_2$

$a_1 a_3 = (a_2)^2 \rightarrow (4)^2 = 16 \Rightarrow a_1 a_3 = 16$

$\left.\begin{array}{l} a_1+a_3=17 \\ a_1 a_3 = 16 \end{array}\right\} \begin{array}{l} a_1 = 1 ,\ a_3 = 16 \rightarrow q = 4 \rightarrow \times \\ a_1 = 16 ,\ a_3 = 1 \rightarrow \boxed{q = \frac{1}{4}} \checkmark \end{array}$

حسابی

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$(x^2+4)(x^2-2) = 4x^2 = x^4+2x^2-8 \Rightarrow x^4-2x^2-8=0$

$(x^2-4)(x^2+2) \Rightarrow x^2=4 \Rightarrow x=-2,\ x=2$

$x^2=-2 \Rightarrow x=\sqrt{-2}$ ✗

$x=-2 \rightarrow a_n = 8, -4, 2 \rightarrow$ ✗ نزولی نیست

$x=2 \rightarrow a_n = 8, 4, 2$

$\Rightarrow a_n = 8\left(\frac{1}{2}\right)^{n-1} \rightarrow \boxed{S_\infty = \frac{8}{1-\frac{1}{2}} = 16}$

۳

$1+q+q^2+q^3+q^4 = \frac{121}{81}$

$243 = a \downarrow$

$a(1+q+q^2+q^3+q^4) = a+aq+aq^2+aq^3+aq^4 = a_1+a_2+a_3+a_4+a_5 = \frac{121}{81} \times 243 \Big\} \boxed{S_5 = 121\times 3 = 363}$

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$d = \frac{64-1}{6} = \frac{63}{6} = 10/5 \rightarrow 1+3d = 1+31/5 = 32/5 \rightarrow A$

$q = \frac{64}{1} = a^6 \rightarrow a=2 \rightarrow 1\times(q)^3 = 1\times 8 = 8 = B$

$\boxed{A+B = 32/5 + 8 = 40/5}$

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$a_n = -24, \frac{95}{4}, \dots \Rightarrow a_n = -24 + (n-1)\frac{1}{4}$

$-\frac{96}{4} \xrightarrow{+\frac{1}{4}}$

$a_{101} = -24 + 100\times\frac{1}{4} = 25-24 = 1$

$t_n = 128, a_2, \dots$

$t_8 = 1 = aq^7 \Rightarrow 128q^7 = 1 \Rightarrow q^7 = \frac{1}{128} \Rightarrow \boxed{q = \frac{1}{2}}$

$$\left.\begin{array}{l} a_3 \to t_1 \\ a_7 \to t_2 \\ a_9 \to t_3 \end{array}\right\} \quad a_3 \times a_9 = (a_7)^2 \Rightarrow \underbrace{(a+2d)(a+8d)}_{a^2+10ad+16d^2} = a^2+36d^2+12ad$$

$$20d^2+2ad=0$$

$$2d(10d+a)=0 \to d=0 \;;\; a+10d=0 \to a=-10d$$

$a=-10d$:

$$\left.\begin{array}{l} t_1 = a_3 = -10d+2d = -8d \\ t_2 = a_7 = -10d+6d = -4d \\ t_3 = a_9 = -10d+8d = -2d \end{array}\right\} \quad q = \frac{-8d}{-4d} = \boxed{2}$$

$d=0$:

$$\left.\begin{array}{l} t_1 = a_3 = a \\ t_2 = a_7 = a \\ t_3 = a_9 = a \end{array}\right\} \quad \boxed{q=1}$$

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$$\left.\begin{array}{l} a_2 = t_x \\ a_4 = t_{x+1} \\ a_8 = t_{x+2} \end{array}\right\} \quad a_2 \times a_8 = (a_4)^2 \Rightarrow \underbrace{(a+d)(a+7d)}_{a^2+8ad+7d^2} = a^2+9d^2+6ad$$

$$2d^2-2ad=0 \Rightarrow 2d(d-a)=0 \to d=0 \;;\; d-a=0 \Rightarrow a=d$$

$$\left.\begin{array}{l} t_x = a_2 = d+a = 2d \\ t_{x+1} = a_4 = a+3d = 4d \\ t_{x+2} = a_8 = a+7a = 8a \end{array}\right\} \quad q = \frac{4a}{2a} = 2 \to t_{10} = \frac{1}{4}(2)^9 = \boxed{128}$$

$$\left.\begin{array}{l} t_x = a_2 = a \\ t_{x+1} = a_4 = a \\ t_{x+1} = a_8 = a \end{array}\right\} \quad q=1 \quad \boxed{t_{10} = \frac{1}{4}}$$

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$$\left.\begin{array}{l} 3t_2 \to a_x \\ 2t_3 \to a_{x+1} \\ t_4 \to a_{x+2} \end{array}\right\} \quad 3t_2+t_4 = 2(2t_3) \Rightarrow \underbrace{3aq+aq^3}_{q(3a+aq^2)} = 4aq^2 \to 3a+aq^2 = 4aq \Rightarrow aq^2-4aq+3a=0$$

$$a(q^2-4q+3)=0$$

$$a(q-3)(q-1)=0$$

$$a=0 \;,\; q=3 \;,\; q=1$$

غیر ثابت

$$\boxed{3=q}$$

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$$a_n = 2, \frac{7}{4}, \dots \to \frac{8}{4}, \frac{7}{4}, \dots \Rightarrow a_n = 2+(n-1)\frac{-1}{4}$$

$$-\frac{1}{4}$$

$$q = \frac{\frac{1}{4}-\frac{19}{4}}{\frac{5}{4}-\frac{19}{4}} = \frac{1-19}{5-19} = \boxed{\frac{9}{7}}$$

$$\left.\begin{array}{l} a_4 = 2+(3)\left(-\frac{1}{4}\right) = \frac{5}{4} = t_x \\ a_8 = 2+(7)\left(-\frac{1}{4}\right) = \frac{1}{4} = t_{x+1} \\ a_{13} = 2+12\left(-\frac{1}{4}\right) = -1 = t_{x+2} \end{array}\right\} \to \left(\frac{5}{4}+x\right)(-1+x) = \left(\frac{1}{4}+x\right)^2$$

$$x^2+\frac{1}{4}x-\frac{5}{4} = x^2+\frac{1}{2}x+\frac{1}{16}$$

$$\Rightarrow -\frac{1}{4}x-\frac{19}{16}=0 \Rightarrow \frac{1}{4}x = -\frac{19}{16} \Rightarrow x = -\frac{\frac{19}{16}}{\frac{1}{4}} = -\frac{19}{4}$$

عددی که [illegible]

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$$\left.\begin{array}{l} t_1 = a_1 \\ t_2 = a_2 \\ t_3 = a_{10} \end{array}\right\} \quad a_1 a_{10} = (a_2)^2 \to a(a+9d) = (a+d)^2 \Rightarrow a^2+9ad = a^2+d^2+2ad \Rightarrow \underbrace{7ad-d^2}=0$$

$$d(7a-d)=0 \to \boxed{d=0} \;;\; d=7a \Rightarrow a=\frac{1}{7}d$$

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$$a_1+a_2+a_{10} = 73 \Rightarrow a+a+d+a+9d = 73 = 3a+10d$$

$$\left.\begin{array}{l} a = \frac{1}{7}d \\ 3a+10d = 73 \end{array}\right\} \quad \frac{3}{7}d+10d = 73 \Rightarrow \frac{73}{7}d = 73$$

$$\boxed{\Rightarrow d = \frac{1}{7}}$$