

$$a_1 + ar + ar^2 = 21 \rightarrow a_1(1+r+r^2) = 21 \rightarrow \frac{a_1}{r} + a_1 + ar = 21 \xrightarrow{\times r} a_1 + ar + ar^2 = 21r$$

$$a_1 \times a_1 r \times a_1 r^2 = a_1^3 r^3 = 4^3 \rightarrow ar = \sqrt[3]{4^3} = 4 \rightarrow a = \frac{4}{r}$$

$$\rightarrow (r^2 - 14r + 4 = 0 \rightarrow r = \frac{14 \pm \sqrt{14^2 - 4 \times 4}}{2} = \frac{14 \pm 12}{2} = \begin{cases} 1 \\ \frac{1}{4} \end{cases}$$

✓ r نیز جواب است پس $r = \frac{1}{4}$ است

$$b^2 = ac \rightarrow (2x)^2 = (x^2 - 2)(x^2 + 2) \rightarrow 4x^2 = x^4 + 2x^2 - 4 \rightarrow x^4 - 2x^2 - 4 = 0$$

$$\xrightarrow{x^2 = z} z^2 - 2z - 4 = 0 \Rightarrow z = x^2 = 1 \pm \sqrt{5} \rightarrow x = \sqrt{1 + \sqrt{5}}, x^2 - 2 = \sqrt{5}, 2x = 2\sqrt{1 + \sqrt{5}}$$

$$x^2 + 2 = 4 + 2\sqrt{5} \rightarrow q = \frac{a_r}{a_1} = \frac{2x}{x^2 - 2} = \frac{2\sqrt{1 + \sqrt{5}}}{\sqrt{5}} \rightarrow ?$$

$$(u^2 - 4)(u^2 + 2) = 0 \rightarrow u = \pm 2$$

$$\begin{cases} n = 2 - 1, 4, 2 \checkmark \\ n = -2 - 1, -4, 2 \times \end{cases}$$

$$S_n = \frac{n(1 - (\frac{1}{4})^n)}{\frac{1}{4}} = \frac{12 \times 4}{1} = 48$$

$$1 + q + q^2 + q^3 + \dots = \frac{121}{11}$$

$$\frac{a(1 + q + q^2 + q^3 + \dots)}{a_1 + a_2 + \dots + a_n} = \frac{121}{11} a = \frac{121}{11} \times \frac{1}{4} = 2.75 \checkmark$$

$$1, -, -, \frac{a}{r}, -, -, 4r$$

$$r^2 a = 4r + 1 = 4 \Rightarrow a = \frac{4}{r} = 2r, 2$$

$$1, -, -, \frac{b}{r}, -, -, 4r$$

$$b^2 = 1 \times 4r = 4r \Rightarrow b = \sqrt{4r} = 2$$

$$\frac{b}{r} = -2 \Rightarrow B = -4$$

$$a + b = 2r, 2 + 2 = 4r, 2 \checkmark$$

$$A + B = 2r, 2$$

$$-\frac{1}{r}, -\frac{1}{r}, \dots$$

$$\xrightarrow{+\frac{1}{r}} d$$

$$a_{101} = a_1 + 100d = -\frac{1}{r} + \left(\frac{100 \times \frac{1}{r}}{r}\right) = 1$$

$$121, a_1, \dots$$

$$a_n = 1 \quad ar^2 = 2^2 \times r^2 = 1 \rightarrow r = \frac{1}{2} \checkmark$$

$$a_r, a_r, a_1 \rightarrow (a+rd), (a+rd), (a+rd) \rightarrow b^r = ac \rightarrow (a+rd)^r = (a+rd)(a+rd)$$

$$\rightarrow a^r + 12ad + 24d^2 = a^r + 10ad + 14d^2 \rightarrow 2ad + 10d^2 = 0 \rightarrow 2d(a+5d) = 0$$

$$a+5d = 0$$

$$d = -\frac{a}{5} \checkmark$$

چونکه مقدار ثابت نیست $\rightarrow 0 = 2ad$ $\rightarrow 0 = a+5d$ $\rightarrow$ بیسونه صفر هم باشه $\checkmark$

$$u+d, a+rd, a+rd \rightarrow (a+rd)^r = (a+d)(a+rd) = a^r + 4ad + 4d^2 = a^r + 10ad + 14d^2$$

$$\rightarrow -2ad + 10d^2 = 0 \rightarrow 2d(-a+5d) = 0 \rightarrow d-a=0 \rightarrow d=a \rightarrow \frac{1}{r} = \frac{1}{r}$$

$$\frac{1}{r}, 1, 2 \rightarrow r=2 \quad a_{10} = a_1 \times r^9 = \frac{1}{r} \times 2^9 = \frac{1}{2} \times 512 = 256$$

جمله اول $\frac{1}{r}$ است! (سوال)

چونکه مقدار ثابت نیست $\rightarrow 2d-a=0$ $\rightarrow 2d=0$

$$rar, rar^r, ar^r \rightarrow rb = a+c \rightarrow r(rar^r) = rar + ar^r \rightarrow rar^2 = r+r^r \rightarrow r^r - rr + r = 0$$

$$r = \frac{r \pm \sqrt{14-12}}{2} = \begin{cases} 3 \\ 1 \end{cases} \checkmark \rightarrow \text{نیست}$$

$$a_1 = 2 \quad a_r = \frac{V}{r} \quad a_r - a_1 = \frac{V}{r} - 2 = -\frac{1}{r} \quad a_r = 2 + (r-1)\left(-\frac{1}{r}\right) = \frac{2}{r} \quad a_8 = \frac{1}{4}$$

$$a_{13} = -1 \quad (a_8 + K), (a_8 + K), (a_{13} + K) \rightarrow \left(\frac{2}{r} + K\right) \times \left(\frac{1}{r} + K\right) \times (K-1) \rightarrow$$

$$\rightarrow \frac{K}{r} + K^2 = -\frac{2}{r} - 1 + \frac{2K}{r} + K^2 + \frac{1}{14} \rightarrow \frac{K}{r} - \frac{K}{r} = -\frac{2}{r} - \frac{1}{14} \rightarrow \frac{K}{r} = -\frac{21}{14}$$

$$\rightarrow K = -\frac{21}{r} \checkmark \rightarrow d = (14 - 21/r) \div (-1 - 21/r) = (-5) \div \left(\frac{r-21}{r}\right) = \frac{5}{r}$$

این تقسیم این نمیشه!! $\rightarrow$ به این تقسیم کردی! $\rightarrow$ جواب $\rightarrow$ $d = \frac{5}{r}$ $\rightarrow$ $d = -\frac{21}{r}, -5, -\frac{21}{r}$ $\rightarrow$ $d = \frac{5}{r}$

$$u+ad^r+ad^4 = Vr \quad a+d = ad^r \quad a+9d = ad^4$$

$$\rightarrow d = ad^r - a \rightarrow a(q^r - 1) \rightarrow 9d = ad^4 - a \rightarrow a(q^4 - 1)$$

$$9d = a(q^4 - 1) \rightarrow 9 \times a(q^r - 1) = a(q^4 - 1) \rightarrow 9q^r - 9 = q^4 - 1 \rightarrow q^4 - 9q^r + 8 = 0$$

$$t = q^r \rightarrow t^2 - 9t + 8 = 0 \rightarrow (t-1)(t-8) = 0 \rightarrow t = \begin{cases} 1 \\ 8 \end{cases} \rightarrow q^r = 1 \vee q^r = 8 \rightarrow q = \begin{cases} 1 \\ 2 \end{cases} \checkmark$$

$$a+ad^r+ad^4 = Vr \rightarrow a(1+q^r+q^4) = Vr \rightarrow a(1+8+64) = Vr \rightarrow a \cdot 73 = Vr \rightarrow a = \frac{Vr}{73}$$

$$d = a(q^r - 1) = 1(8-1) = 7 \checkmark$$