پارسا ضیائی — تکلیف شماره ۱۵ دهم سپهر A

Subject: Date

۱- $a_1\left(\frac{q^3-1}{q-1}\right)=21 \Rightarrow a(q^2+q+1)=21 \Rightarrow a_1q^2+a_1q+a_1=21 \Rightarrow 4q+4+\frac{4}{q}=21$

$a_1\times a_1q\times a_1q^2=64 \Rightarrow a_1^3q^3=64 \Rightarrow a_1q=4$

$\Rightarrow 4q^2+4q+4=21q \Rightarrow 4q^2-17q+4=0$

$q^2-17q+16=0$

$(q-1)(q-16) \to q \begin{cases} \frac{1}{4} \ \times \\ \frac{16}{4}=4 \ \checkmark \end{cases}$

۲- $x^2+4,\ 2x,\ x^2-2 \to (x^2+4)(x^2-2)=(2x)^2 \Rightarrow x^4+2x^2-8=4x^2$

$\Rightarrow x^4-2x^2-8=0 \to (x^2-4)(x^2+2)=0 \Rightarrow x^2 \begin{cases} -2 & \text{غیرممکن} \\ 4 \Rightarrow x=\pm 2 \end{cases}$

$a_n \begin{cases} 8,4,2 \\ 8,-4,2 \end{cases} \xRightarrow{q>0} a_n=8,4,2 \to q=\frac{1}{2}$

$S_7=8\left(\frac{1-(\frac{1}{2})^7}{1-\frac{1}{2}}\right)=8\times\frac{\frac{127}{128}}{\frac{1}{2}}=\cancel{8}\times\frac{127}{64}=\frac{127}{8}$

۳- $1+q+q^2+q^3+q^4=\frac{121}{81}$ $a_1=243$

$S_5=a_1+a_1q+a_1q^2+a_1q^3+a_1q^4=a_1(1+q+q^2+q^3+q^4)=\cancel{243}^{3}\times\frac{121}{\cancel{81}}=363$

۴- $d=\frac{64-1}{6}=10.5 \to a_n=1, 11.5, 22, \underbrace{32.5}_{A}, 43, 53.5, 64$

$q^6=\frac{64}{1}\Rightarrow q=\pm 2 \Rightarrow b_n=\begin{cases} 1,2,4,\boxed{8},16,32,64 \\ 1,-2,4,\boxed{-8},16,-32,64 \end{cases}$ B

$A+B \begin{cases} 32.5+8=40.5 \\ 32.5-8=24.5 \end{cases}$

۵- $-24, -\frac{95}{4}, \dots \Rightarrow a_n=\frac{-96}{4}, \frac{-95}{4}, \dots \Rightarrow a_n=\frac{n-97}{4} \Rightarrow a_{101}=\frac{4}{4}=1$

$128, a_2, a_3, a_4, a_5, a_6, a_7, 1, \dots$ ($\div 2$, $\div 2$, $\div 2$, $\div 2$, $\div 2$, $\div 2$, $\div 2$) $\Rightarrow q^7=\frac{1}{128} \Rightarrow q=\frac{1}{2}$

$\underbrace{(a+2d),(a+6d),(a+8d)}_{\times} \to a^2+10ad+16d^2=a^2+12ad+36d^2 \Rightarrow 20d^2+2ad=0$

$\Rightarrow 2d(10d+a)=0 \Rightarrow d \begin{cases} 0 \\ \frac{-a}{10} \end{cases}$

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$a+d,\ a+3d,\ a+7d \rightarrow \underbrace{a^2+8ad+7d^2}_{\times} = a^2+6ad+9d^2$ — ۷

$\Rightarrow 2d^2-2ad=0 \Rightarrow 2d(d-a)=0 \Rightarrow d \begin{cases} 0 & \times \\ a & \checkmark \end{cases}$

$a+d=2a=\frac{1}{4} \Rightarrow a=\frac{1}{8}$
$a+3d=4\times\frac{1}{8}=\frac{1}{2}$
$\Rightarrow q=\dfrac{\frac{1}{2}}{\frac{1}{4}}=2 \Rightarrow a_{10}=a_1q^9=\frac{1}{4}\times 2^9=128$

$3a_1q,\ 2a_1q^2,\ a_1q^3 \rightarrow 3a_1q+a_1q^3=2\times 2a_1q^2$ — ۸

$\Rightarrow \cancel{a_1q}(3+q^2)=\cancel{4a_1q^2} \Rightarrow 3+q^2=4q \Rightarrow q^2-4q+3=0$
$4q$

$\Rightarrow (q-3)(q-1)=0 \Rightarrow q \begin{cases} 1 & \times \\ 3 & \checkmark \end{cases}$

$2,\ \frac{7}{4},\ \dots \Rightarrow a_n=\frac{9-n}{4} \Rightarrow \begin{cases} a_4=\frac{5}{4} \\ a_8=\frac{1}{4} \\ a_{13}=\frac{-4}{4} \end{cases}$ — ۹

$b_n=\frac{5}{4}+x,\ \frac{1}{4}+x,\ \frac{-4}{4}+x \rightarrow \underbrace{x^2+\frac{x}{4}-\frac{5}{4}}_{\times}=\cancel{x^2}+\frac{x}{2}+\frac{1}{16}$

$\Rightarrow 4x-20=8x+1 \Rightarrow 4x=-21 \Rightarrow x=\frac{-21}{4}$

$b_n=\frac{5}{4}+x,\ \frac{1}{4}+x,\ \frac{-4}{4}+x \rightarrow -4,\ -5,\ \frac{-25}{4} \rightarrow q=\frac{-5}{-4}=1{,}25$

$a_1+\underbrace{a_1q^3}_{\times q^3}+\underbrace{a_1q^6}_{\times q^3}=73 \rightarrow a_1(1+q^3+q^6)=73$ — ۱۰

دنباله حسابی: $a_1 \quad a_1q^3 \quad \dots \quad a_1q^6$
$(1) \quad (2) \quad (10)$

$\rightarrow \begin{cases} d=a_1q^3-a_1 \\ d=\dfrac{a_1q^6-a_1q^3}{8} \end{cases} \Rightarrow \cancel{a_1(q^3-1)}=\dfrac{\cancel{a_1}q^3\cancel{(q^3-1)}}{8}$

$\Rightarrow \frac{q^3}{8}=1 \Rightarrow q=2$

$a_1(\overbrace{1+q^3+q^6}^{q=2})=73 \Rightarrow a_1(73)=73 \Rightarrow a_1=1$

$d=a_1q^3-a_1 \Rightarrow d=8-1=7$