

$$a_1 + a_r = 21 \Rightarrow a_1 + a_1 r^3 = 21 \Rightarrow a(1+r^3) = 21$$

$$a_r + a_{r^3} = 12 \Rightarrow a_1 r + a_1 r^3 = 12 \Rightarrow ar(1+r^3) = 12$$

$$a = \frac{12}{r(1+r)} \Rightarrow 3(1+r^3) = 7r(1+r) \Rightarrow 3r^3 - 7r^2 - 7r + 3 = 0$$

$$r = \frac{1}{3}, r = 3 \quad \left\{ \begin{array}{l} 1, 3, 9, 27 \\ 27, 9, 3, 1 \end{array} \right.$$

بزرگترین عدد 27 می شود. ✓

$$a + ar + ar^3 = 13$$

$$a \cdot ar \cdot ar^3 = a^3 r^3 = 27 \Rightarrow ar = \sqrt[3]{27} = 3$$

$$a + 3 + ar^3 = 13 \Rightarrow a + ar^3 = 10 \Rightarrow a_1 = 1 \underline{9}, a_3 = 1 \underline{9}$$

$$1, 3, 9 \quad \checkmark \quad 9, 3, 1 \quad \checkmark$$

$$1, 4, 16 \Rightarrow q = 3, a = 1$$

$$S_n = a \left(\frac{q^n - 1}{q - 1} \right) \Rightarrow S_{10} = \frac{1(3^{10} - 1)}{3 - 1} = \boxed{59049} \quad (\text{الف})$$

$$P_n = a_1^n r^{\frac{n(n-1)}{2}} \Rightarrow P_{10} = 1^{10} \times 3^{\frac{10(9)}{2}} \Rightarrow \boxed{1^9 \times 3^{45}} \quad \checkmark$$

$$a_n = aq^{n-1} \Rightarrow b_n = a_{n+1} - a_n \Rightarrow b_n = a_{n+1} - a_n$$

$$\Rightarrow b_n = a_{n+1} - a_n = aq^n - aq^{n-1} = aq^{n-1}(q-1).$$

$$\Rightarrow b_n = a(q-1)q^{n-1}, \quad b_{n+1} = a(q-1)q^n$$

$$\frac{b_{n+1}}{b_n} = \frac{a(q-1)q^n}{a(q-1)q^{n-1}} = \boxed{q} \quad \checkmark$$

$$a_n = a_1 + (n-1)d$$

$$S_n = a_1 + a_2 + a_3 + \dots + a_n$$

$$S_n = a_1 + a_2 + \dots + a_n \quad (\text{الف})$$

$$S_n = a_n + a_{n-1} + \dots + a_1$$

$$S_n = a + ar + ar^2 + \dots + ar^{n-1} \quad (\text{ب})$$

$$rS_n = ar + ar^2 + \dots + ar^n$$

$$S_n - rS_n = a - ar^n$$

$$\Rightarrow (1-r)S_n = a(1-r^n) \Rightarrow S_n = \frac{a(1-r^n)}{1-r} \quad \checkmark \quad \text{اثبات 1}$$

$$rS_n = (a_1 + a_n) + (a_2 + a_{n-1}) + \dots + (a_n + a_1).$$

$$\Rightarrow rS_n = n(a_1 + a_n) \Rightarrow S_n = \frac{n}{r} (a_1 + a_n).$$

$$\Rightarrow S_n = \frac{n}{r} (2a_1 + (n-1)d). \quad \checkmark \quad \text{اثبات 2}$$