

سوال ۱

الف: $\frac{1}{r} \times r^9 = r^{\wedge} = 206$

ب: $\frac{a_{17}}{a_{13}} = \frac{\frac{1}{r} \times r^{17}}{\frac{1}{r} \times r^{13}} = r^4 = 16$

ج: $a_4 = \frac{1}{r} \times r^4 = \frac{1}{r} \times 1 = \frac{1}{r}$ و $a_{10} = \frac{1}{r} \times r^{10} = 206 \rightarrow$

د: $\frac{1}{r} \times r^{n-1} = 121 \Rightarrow r^{n-1} = 206 \quad r=2 \Rightarrow r^{n-1} = 206 \Rightarrow n=9$

سوال ۲

$\frac{a_1 q^v}{a_1 q^k} = \frac{96}{12} = 8 \Rightarrow q = 8 \Rightarrow q = 2$

$a_1 \times 16 = 12 \Rightarrow a_1 = \frac{3}{4}$
 $\Rightarrow a_{10} = \frac{3}{4} \times 2^9 = \frac{3}{4} \times 512 = 384$

سوال ۳

$P_8 = 243$

الف: $(a_n)^8 = 243 \Rightarrow a_n = \sqrt[8]{243} = 3$

ب: $\sqrt{(a_1 \cdot a_8)^8} = 243 \Rightarrow (a_1 \cdot a_8)^8 = 243^2 \Rightarrow a_1 \cdot a_8 = 9$

سوال ۴

$(\sqrt{5})^r = r^a \times r^b \Rightarrow 32 = r^{a+b} \Rightarrow a+b=5$

$r^1 \cdot \sqrt{5} \cdot r^4 \cdot \dots \rightarrow$ دنباله هندسی با $a=1$ و $b=4 \rightarrow 1, 2, 4, 8, 16, 32$

$\frac{r+1}{r} - \frac{8}{r} = 2$

سوال ۵

$a_1 q^r = 1-x$
 $a_1 q^s = x$
 $a_1 q^{10} = x+1$

$\Rightarrow x^2 = 1-x^2 \Rightarrow 2x^2 = 1 \Rightarrow x^2 = \frac{1}{2} \Rightarrow x = \pm \sqrt{\frac{1}{2}} \Rightarrow \pm \frac{1}{\sqrt{2}}$

$\frac{a_1 q^{10}}{a_1 q^r} = \frac{x+1}{1-x} = q^8 \Rightarrow x^2 = (x+1)(1-x) + \frac{1}{2}$

سوال ۶

$a_1 + a_8 = 21$
 $a_1 + a_1 q^7 = 21$
 $a_1 (1+q^7) = 21$
 $a_1 (1) = 21 \Rightarrow a_1 = 1$

$a_4 + a_9 = 12$
 $a_1 q + a_1 q^8 = 12$
 $a_1 q (1+q^7) = 12$
 $\Rightarrow a_1 = 1$

$\frac{a_4 (1+q^7)}{a_1 q (1+q^7)} = \frac{21}{12} \Rightarrow q = 3$
 $a_4 = 27 \quad a_9 = 3 \quad a_{10} = 9$

$$(n \text{ و سلی})^n = P_n \Rightarrow (a_p)^n = 27 = a_p = 3 \Rightarrow a_1 \cdot 3 \cdot a_p = 27$$

$$\Rightarrow a_1 \cdot a_p = 9$$

$$\Rightarrow a_1 = 1$$

$$a_p = 3$$

$$a_p = 9$$

$$\Rightarrow 1, 3, 9, \dots$$

الف: $(S_{10}) = 2 \left(\frac{3^{10} - 1}{3 - 1} \right) \Rightarrow 2 \left(\frac{3^{10} - 1}{2} \right) = 3^{10} - 1$

ب: $(P_{10}) = (a_1 \cdot a_{10})^5 \Rightarrow (a_1 \times a_{10} \times q^9)^5 = (1 \times 3^9)^5$

$\frac{a}{q}, a, aq, aq^2, \dots$

$$\frac{a}{q} - a = \frac{a(1-q)}{q} \quad | \quad a - aq = a(1-q) \quad | \quad aq - aq^2 = aq(1-q)$$

دنباله $\rightarrow \frac{a(1-q)}{q}, a(1-q), aq(1-q)$

ا. ب. ت $\rightarrow \frac{a(1-q)}{q} \times aq(1-q) = (a(1-q))^2 = \frac{a^2(1-q)^2}{q}$ درسته

قدر نسبت $\rightarrow \frac{a(1-q)}{a(1-q)} = 1$

الف: $S_n = a + a + d + a + 2d + \dots + (a + (n-2)d) + (a + (n-1)d)$

$$= (a + (n-1)d) + (a + (n-2)d) + \dots + a + d + a$$

تساوی

$\Rightarrow 2S_n = (2a + (n-1)d) + (2a + (n-1)d) + \dots + (2a + (n-1)d)$

بی نهایت بار در واقع (n) تا عبارت میاد هم داریم

$\Rightarrow 2S_n = n(2a + (n-1)d)$

$\Rightarrow S_n = \frac{n}{2}(2a + (n-1)d)$

ب: $S_n = a + aq + aq^2 + \dots + aq^{n-1}$

$qS_n = aq + aq^2 + aq^3 + \dots + aq^n$

عندسی

$\rightarrow S_n - qS_n = a - aq^n$

$\rightarrow S_n(1-q) = a(1-q^n) \Rightarrow S_n = \frac{a(1-q^n)}{1-q} \text{ if } q \neq 1$