نام و نام خانوادگی ..... پاسخنامه تشریحی تکلیف شماره ۱۴ ..... کلاس ..... A .....

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الف) $a_{10} = aq^9 = \frac{1}{2} \times 2^{12} = 2^{11}$

ب) $\frac{a_{17}}{a_{12}} = \frac{aq^{16}}{aq^{11}} = q^5 = 2^5 = 16$

ج) $a_4 = aq^3 = \frac{1}{2} \times 8 = 4$

$4 \times 2^{11} = \sqrt{2^{10} \times 2^4} = 32$

د) $a_n = \frac{1}{2} \times 2^{n-1} = 128 \Rightarrow 2^{n-1} = 2^8 \Rightarrow n-1 = 8 \Rightarrow n = 9$

$a = \frac{1}{2}$, $q = 2$

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$a \times 2^6 = 12$

$a = \frac{12}{2^6} = \frac{12}{16} = \frac{3}{4}$

$\frac{a^8}{a^4} = \frac{aq^7}{aq^4} = q^3 = \frac{96}{12} = 8 \Rightarrow q = 2$

$a_{10} = aq^9 = \frac{3}{4} \times 2^9 = 384$

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$a_1 \times a_2 \times a_3 \times a_4 \times a_5 = 243$

$a_1 \times a_5 = a_3 \times a_3$

$a_2 \times a_4 = a_3 \times a_3$

$a_3 \times a_3 \times a_3 \times a_3 \times a_3 = 243$

$(a_3)^5 = 243$

$a_3 = 3$

$a_1 \times a_5 = a_3 \times a_3 = 3^2 = 9$

۴

$2^a \quad 4\sqrt{2} \quad 2^b$

$\frac{a+b}{2} = \frac{5}{2} = 2.5$

$4\sqrt{2} = \sqrt{32}$

$2^a \times 2^b = \sqrt{32} \times \sqrt{32}$

$2^{a+b} = 32 \Rightarrow a+b = 5$

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$a_{13} \times a_{11} = a_{12} \times a_7$

$(x+1)(1-x) = x^2$

$x - x^2 + 1 - x \Rightarrow -x^2 + 1 = x^2$

$2x^2 = 1$

$x^2 = \frac{1}{2}$ $\quad x = \pm \frac{1}{\sqrt{2}} = \pm \frac{\sqrt{2}}{2}$

$x = -\frac{1}{\sqrt{2}}$

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$a + aq^3 = 28$

$aq + aq^2 = 12$

$aq(1+q)$

$a(1+q^3) \to a(28) = 28$, $a = 1$

$1, 3, 9, 27$; $27, 9, 3, 1$; $q = 3$, $q = \frac{1}{3}$

$$\frac{a(1+q^3)}{aq(1+q)} = \frac{(1+q)(1-q+q^2)}{q(1+q)} = \frac{28}{12} = \frac{7}{3}$$

$3 - 3q + 3q^2 = 7q$

$3q^2 - 10q + 3 = 0$

~~$\Delta = b^2 - 4ac = 100 - 36 = 64$~~

$q^2 - 10q + 9 = (q-9)(q-1)$

$q = \frac{9}{3} = 3$, $q = \frac{1}{3}$

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$a + aq + aq^2 = 13$

~~$a(1+q+q^2) = 13$~~

$a \times aq \times aq^2 = 27$

$a^3q^3 = 27$

$(aq)^3 = 27$

$aq = 3 \Rightarrow a = 9,\ q = \frac{1}{3}$; $a = 1,\ q = 3$

$\frac{3}{q} + 3 + 3q = 13$

$\frac{3}{q} + 3q = 10$

$3 + 3q^2 = 10q$

$3q^2 - 10q + 3 = 0$

$\Delta = b^2 - 4ac = 100 - 36 = 64$

$q = \frac{10 \pm \sqrt{64}}{6} \Rightarrow q = 3$, $q = \frac{1}{3}$

a_1	a_2	a_3
1	3	9
9	3	1

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$a_n = 2, 6, 18, \ldots$

$a = 2$, $q = 3$

$$S_{10} = 2\left(\frac{3^{10} - 1}{3 - 1}\right) = 3^{10} - 1 = 59048$$

$a_{10} = 2 \times 3^9 = 39366$

$(a_1 \times a_{10})^5 = (2 \times 2 \times 3^9)^5 = (4 \times 3^9)^5 = 4^5 \times 3^{45} = 2^{10} \times 3^{45}$

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$a,\ aq,\ aq^2,\ aq^3$; $a(q-1),\ aq(q-1),\ aq^2(q-1),\ aq^3(q-1), \ldots$

$(aq - a), (aq^2 - aq), (aq^3 - aq^2)$

$aq - a = a(q-1)$

$aq^2 - aq = aq(q-1)$

$aq^3 - aq^2 = aq^2(q-1)$

$$\frac{aq^n(q-1)}{aq^{n-1}(q-1)} = \frac{q^n}{q^{n-1}} = q$$

چون نسبت جمله‌ی بعدی به جمله‌ی قبلی همیشه ثابت است دنباله هندسی است و قدر نسبت آن q همان قدر نسبت قبلی خواهد بود

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الف)

$S_n = a + aq + aq^2 + \cdots aq^{n-1}$

$qS_n = aq + aq^2 + aq^3 + \cdots aq^n$

$S_n - qS_n = (a + aq + aq^2 + \cdots aq^{n-1}) - (aq + \cdots aq^n)$

$S_n - qS_n = a - aq^n$

$S_n(1-q) = a(1-q^n)$

$S_n = \frac{a(1-q^n)}{(1-q)}$, $q \neq 1$, $= \frac{a(q^n - 1)}{q - 1}$

ب)

$a,\ a+d,\ a+2d, \ldots$

$S_n = a + (a+d) + (a+2d) + \cdots (a + (n-1)d)$

$S_n = (a + (n-1)d) + (a + (n-2)d) + \cdots + a$

$2S_n = [a + a + (n-1)d] + [a + (a + (n-2)d] + \cdots [a + a]$

$2S_n = n(2a + (n-1)d) \Rightarrow S_n = \frac{n}{2}(2a + (n-1)d)$