نام و نام خانوادگی ..... پاسخنامه تشریحی تکلیف شماره ..۱۴.. کلاس ..۱۰A.....

۱

الف)
$\frac{1}{4}, \frac{1}{2}, 1, 2, \dots$
$\frac{\times 2}{\times 2}$ $\Rightarrow q = 2$
$a_1 q^{n-1} \rightarrow \frac{1}{4} \times 2^{10-1} = 2^8 = 256$

ب)
$a_{17} = a_{13} q^4$
$\frac{a_{17}}{a_{13}} = q^4 \rightarrow 2^4 = 16$

ج)
$a_{10} = a_1 q^9$
$a_4 = a_1 q^3$
$\frac{\frac{1}{4} \times 2^3 + \frac{1}{4} \times 2^9}{2} = \frac{260}{2} = 135$

د)
$a_1 q^{n-1} = 2^7$
$1248 = 2^7$
$2^{-1} \times 2^{n-1} = 2^7$
$2^{n-1} = 2^8$
$n = 9$

۲

$a_5 = 12$
$a_8 = 96 \rightarrow a_5 q^3$
$12 \times q^3 = 96$
$q^3 = 8$
$q = \sqrt[3]{8} = 2$
$a_{10} = a_8 q^2 \rightarrow 96 \times 2^2 = 384$

۳

الف)
$a_1 \times a_1 q \times a_1 \times q^2 \times a_1 \times q^3 \times a_1 \times q^4 = a_1^5 q^{10} = 243$
$a_3 \Rightarrow P_n = (a_3)^5 = 243$ $a_3^5 = 3^5$ $a_3 = 3$

ب)
$P_n = \sqrt{(a_1 a_n)^n} \rightarrow 243 = \sqrt{(a_1 a_5)^5} \rightarrow 3^5 = (a_1 a_5)^5 \rightarrow a_1 a_5 = 9$

۴

$2^a, 4\sqrt{2}, 2^b \rightarrow 2^{a+b}$
$2^5 = 16 \times 2$
$a + b = 5$

$\frac{a+b}{2} = 2.5$

۵

$a_1 q^{10} = \frac{x+1}{x} = \frac{x}{x+1} \rightarrow x^2 = 1 - x^2$ $x^2 = \frac{1}{2}$ $x = \pm\sqrt{\frac{1}{2}}$

(۱) $a_2 = 1 - \frac{1}{\sqrt{2}}$ ، $a_7 = \frac{1}{2}$ ، $a_{11} = 1 + \frac{1}{\sqrt{2}}$ $q^2 = \frac{a_7}{a_2} = \frac{\frac{1}{\sqrt{2}}}{1 - \frac{1}{\sqrt{2}}} > 0$

(۲) $a_2 = 1 + \frac{1}{\sqrt{2}}$ ، $a_7 = -\frac{1}{2}$ ، $a_{11} = 1 - \frac{1}{\sqrt{2}}$ $q^2 = \frac{-\frac{1}{\sqrt{2}}}{1 + \frac{1}{\sqrt{2}}} < 0$

$x = \frac{1}{\sqrt{2}}$

۶

$a_1 + a_4 = 28$ $\quad a_1 = 1 - a_2 = 8^2 - 8^2 = 9 - a_4 = 27$

$a_2 + a_3 = 12$ $\longrightarrow$

$\boxed{a_4 = 27}$

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$a_1 + a_2 + a_3 = 13$ $\rightarrow$ $a_1 + a_1 q + a_1 q^2 \rightarrow a_1(1 + q + q^2) = 13$

$a_1 \times a_2 \times a_3 = 27$ $\rightarrow$ $a_1 \times a_1 q \times a_1 q^2 \rightarrow a^3 \times q^3 = 27$

$q = 3$
$a_1 = 1$

1	3	q	q^2
a_1	a_2	a_3	

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الف)

$3, 9, 27, \dots$ $\quad q = 3$

$S_{10} = \frac{a_1(q^n - 1)}{q - 1} = \frac{3 \times (3^{10} - 1)}{3 - 1} = 3^{10} - 1 = 59049 - 1$

$P_n = \left(\sqrt{a_1 \cdot a_n}\right)^n \rightarrow \left(\sqrt{3 \times 3 \times 3^9}\right)^{10} \rightarrow \left(3\sqrt{3^9}\right)^{10}$

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$1, 2, 4, 8, 16, \dots$ $\quad q = 2$

$\rightarrow$ نسبت

$-1, -2, -4, -8$

$a_1, a_1 q, a q^2 \rightarrow a_1 - a_1 q, a_1 q - a q^2$

$a_1(1 - q), a_1 q(1 - q)$

$\times q$

۱۰

الف)

$a, a+d, a+2d, a+3d, \dots$ $\quad S_n = a + (a+d) + (a+2d) + \dots + (a+(n-1)d)$

$S_n = a + (a+d) + (a+2d) + \dots + (a+(n-1)d) = (a + (n-1)d) + (a + (n-2)d) + \dots + a$

$2S_n = [a + (a+(n-1)d)] + [(a+d) + (a+(n-2)d)] + \dots \quad 2S_n = (2a + (n-1)d)n$

$S = (2a + (n-1)d)\frac{n}{2}$

ب)

$S_n = a + aq + aq^2 + \dots = aq^{n-1} \Rightarrow qS_n = aq + aq^2 + aq^3 + \dots + aq^n$

$(a + aq + aq^2 + \dots + aq^{n-1}) - (aq + aq^2 + \dots + aq^n) \rightarrow qS - S$

$aq^n - a = S(q - 1)$

$S_n = \frac{a(1 - q^n)}{1 - q} = \frac{a(q^n - 1)}{q - 1}$