

الف) $a_n = \frac{1}{r} \times r^{n-1} \rightarrow a_{10} = \frac{1}{r} \times r^9 = r^8 = \boxed{256}$

ب) $\frac{a_{14}}{a_{12}} = \frac{a q^{14}}{a q^{12}} = q^2 = r^2 = \boxed{16}$

ج) $x = \sqrt{a_{12} a_{10}} = \sqrt{a_1^2 \times q^{12}} = a_1 q^6 = \frac{1}{r} \times r^6 = r^5 = \boxed{32}$

د) $\frac{1}{r} \times r^{n-1} = 128 \Rightarrow r^{n-1} = r^8 \Rightarrow \boxed{n=9}$

$\frac{a_8}{a_5} = \frac{a q^8}{a q^5} = q^3 = \frac{96}{12} = 8 \Rightarrow q = 2$

$a_{10} = a_1 \times q^9 = 96 \times 2^9 = \boxed{3168}$

$\frac{a_8}{q^8} \times \frac{a_8}{q^8} \times a_8 \times a_8 q \times a_8 q^2 = a_8^5 = r^{13} \Rightarrow a_8 = \sqrt[5]{r^{13}} \Rightarrow \boxed{a_8 = 3}$
الف

$a_1 \times a_5 = a_3 \times a_7 = 3 \times 3 = 9$
ب

$r^a, r^{\frac{\omega}{r}}, r^b \Rightarrow r^a \times r^b = (r^{\frac{\omega}{r}})^r \Rightarrow r^{a+b} = r^{\omega} \Rightarrow a+b = \omega$

$a, x, b \Rightarrow r^x = \underbrace{a+b}_{\omega} \Rightarrow x = \frac{\omega}{r} = \boxed{r, \omega}$

$a_v \times a_v = a_r \times a_{11} \Rightarrow x^r = (x+1)(1-x) = 1 - x^2 \Rightarrow r x^r = 1$

$x^r = \frac{1}{r}$

$x = \pm \frac{1}{\sqrt[r]{r}} = \pm \frac{\sqrt[r]{r}}{r}$

$\rightarrow \frac{\sqrt[r]{r}}{r}$
 $\rightarrow -\frac{\sqrt[r]{r}}{r}$

$$\left. \begin{aligned} a + aq^r &= 18 \\ aq + aq^r &= 12 \end{aligned} \right\} \Rightarrow \frac{a + aq^r}{aq + aq^r} = \frac{18}{12} \Rightarrow \frac{a(q^r - 1)}{aq(q + 1)} = \frac{3}{2} = \frac{v}{r}$$

$$\Rightarrow vq = r^2q^r + r - rq \Rightarrow r^2q^r - 1 \cdot q + r = 0 \Rightarrow q^r - 1 \cdot q + 9 = (q - 1)(q - 9)$$

$$\Rightarrow q = \begin{cases} \frac{1}{r} \\ q = 3 \end{cases}$$

$$q = \frac{1}{r} \Rightarrow a + \frac{a}{r^v} = 18 \Rightarrow a = 27$$

$$\Rightarrow (27, 9, 3, 1) \Rightarrow (27) \text{ بزرگترین}$$

$$q = 3 \Rightarrow a + 27a = 18 \Rightarrow a = 1$$

$$\Rightarrow 1, 3, 9, (27) \rightarrow \text{بزرگترین}$$

در هر دو حالت بزرگترین ۲۷ است.

$$\frac{a_r}{q} \times a_r \times a_r q = 27 \Rightarrow a_r^3 = 27 \Rightarrow a_r^3 = 3$$

$$\Rightarrow \frac{r}{q} + 3 + 3q = 13 \Rightarrow 3 + 3q + 3q^r = 13q \Rightarrow r^2q^r - 1 \cdot q + r = 0$$

$$(q - 1)(q - 9) = 0$$

$$\Rightarrow q = \begin{cases} \frac{1}{r} \\ q = 3 \end{cases}$$

$$\Rightarrow a_1, a_2, a_3$$

$$q = 3 \rightarrow 1, 3, 9$$

$$q = \frac{1}{r} \rightarrow 9, 3, 1$$

دو حالت

$$S_{10} = a \left(\frac{q^{10} - 1}{q - 1} \right) = \frac{r^{10} - 1}{r - 1} = \frac{3^{10} - 1}{3 - 1} = 59049 - 1 = \underline{59048}$$

$$P_{10} = (a \times a_1)^{\frac{1}{r}} = (a^r \times a \times q^9)^{\frac{1}{r}} = a^{\frac{1}{r}} \times a^{\frac{r-1}{r}} \times q^{\frac{9}{r}} = 2 \times 3$$

$$a, aq, aq^2, aq^3, \dots, aq^{n-1}$$

$$aq - a, aq^2 - aq, aq^3 - aq^2, \dots, aq^n - aq^{n-1}$$

$$\underbrace{a(q-1)}_{a_1}, \underbrace{aq(q-1)}_{\text{قد رست}}, \underbrace{aq^2(q-1)}_{\text{قد رست}}, \dots, a(q-1)q^{n-1} \rightarrow \text{قد رست هر دو دنباله مساوی است}$$

$q = \text{قد رست}$

$$(a) \quad (a) + (a+d) + (a+2d) + \dots + (a+(n-1)d) = na + \frac{n(n-1)}{2}d = \frac{n}{2}(ra + (n-1)d)$$

$$\Rightarrow S_n = \frac{n}{2}(ra + (n-1)d)$$

$$ب) \quad (a + aq + aq^2 + \dots + aq^{n-1}) \frac{q-1}{q-1}$$

$$= (\cancel{aq} - a + \cancel{aq^2} - \cancel{aq} + \cancel{aq^3} - \cancel{aq^2} + \dots + aq^n - \cancel{aq^{n-1}}) \frac{1}{q-1} = \frac{aq^n - a}{q-1}$$

$$= a \left(\frac{q^n - 1}{q - 1} \right) = S_n$$