

الف) $a_n = \frac{1}{r} \times r^{n-1} \rightarrow a_1 = \frac{1}{r} \times r^9 = r^8 = \boxed{256}$ ✓

ب) $\frac{a_{11}}{a_{13}} = \frac{a_1 q^{10}}{a_1 q^{12}} = q^{-2} = r^{-2} = \boxed{16}$ ✓

ج) $x = \sqrt{a_5 a_1} = \sqrt{a_1^2 \times q^{10}} = a_1 q^5 = \frac{1}{r} \times r^9 = r^8 = \boxed{256}$ ✓

د) $\frac{1}{r} \times r^{n-1} = 128 \Rightarrow r^{n-1} = r^8 \Rightarrow \boxed{n=9}$ ✓

$\frac{a_1}{a_9} = \frac{a_1 q^0}{a_1 q^8} = q^{-8} = \frac{1}{128} \Rightarrow q = 2$

$a_1 = a_9 \times q^8 = 128 \times 2^8 = \boxed{32768}$ ✓

$\frac{a_7}{q^7} \times \frac{a_7}{q} \times a_7 \times a_7 q \times a_7 q^7 = a_7^5 = r^{43} \Rightarrow a_7 = \sqrt[5]{r^{43}} \Rightarrow \boxed{a_7 = 3}$ ✓
الف

$a_1 \times a_9 = a_7 \times a_7 = 3 \times 3 = \boxed{9}$ ✓
ب

$r^a, r^{\frac{\omega}{r}}, r^b \Rightarrow r^a \times r^b = (r^{\frac{\omega}{r}})^r \Rightarrow r^{a+b} = r^{\omega} \Rightarrow a+b = \omega$

$a, x, b \Rightarrow r^x = \underbrace{r^a r^b}_\omega \Rightarrow x = \frac{\omega}{r} = \boxed{r, \omega}$ ✓

$a_v \times a_v = a_r \times a_{11} \Rightarrow x^r = (x+1)(1-x) = 1 - x^2 \Rightarrow r x^r = 1$

$x^r = \frac{1}{r}$

$x = \pm \sqrt[r]{\frac{1}{r}} = \pm \frac{\sqrt[r]{r}}{r}$

خواب و ... $\leftarrow x < 0$ q بیش!

$$\left. \begin{aligned} a + aq^r &= 18 \\ aq + aq^r &= 12 \end{aligned} \right\} \Rightarrow \frac{a + aq^r}{aq + aq^r} = \frac{18}{12} \Rightarrow \frac{a(q^r - 1)}{aq(q + 1)} = \frac{3}{2} \quad (2)$$

$$\Rightarrow 2q = 3q^r + 3 - 3q \Rightarrow 3q^r - 1 \cdot q + 3 = 0 \Rightarrow q^r - 1 \cdot q + 9 = (q - 1)(q - 9)$$

$$\Rightarrow q = \begin{cases} \frac{1}{3} \\ q = 3 \end{cases}$$

$$q = \frac{1}{3} \Rightarrow a + \frac{a}{3} = 18 \Rightarrow a = 27$$

$$\Rightarrow (27, 9, 3, 1) \Rightarrow (27) \text{ بزرگترین}$$

$$q = 3 \Rightarrow a + 27a = 18 \Rightarrow a = 1$$

$$\Rightarrow 1, 3, 9, (27) \rightarrow \text{بزرگترین} \quad \checkmark \text{ در هر دو حالت بزرگترین 27 است.}$$

$$\frac{a_r}{q} \times a_r \times a_r q = 27 \Rightarrow a_r^3 = 27 \Rightarrow a_r^3 = 3 \quad (2)$$

$$\Rightarrow \frac{3}{q} + 3 + 3q = 13 \Rightarrow 3 + 3q + 3q^r = 13q \Rightarrow 3q^r - 1 \cdot q + 3 = 0$$

$$(q - 1)(q - 9) = 0 \quad V$$

$$\Rightarrow q = \begin{cases} \frac{1}{3} \\ q = 3 \end{cases} \Rightarrow a_1, a_2, a_3$$

$$q = 3 \rightarrow 1, 3, 9$$

$$q = \frac{1}{3} \rightarrow 9, 3, 1$$

دو حالت $\checkmark$

$$S_{10} = a \left(\frac{q^{10} - 1}{q - 1} \right) = 3 \left(\frac{3^{10} - 1}{3 - 1} \right) = 3^{10} - 1 = 59049 - 1 = 59048 \quad \checkmark \quad (2)$$

$$P_{10} = (a \times a_1)^{\frac{1}{10}} = (a^r \times a \times q^9)^{\frac{1}{10}} = a^{\frac{1}{10}} \times q^{\frac{9}{10}} = 3^{\frac{1}{10}} \times 3^{\frac{9}{10}} = 3 \quad \checkmark \quad \wedge$$

$$a, aq, aq^2, aq^3, \dots, aq^{n-1}$$

$$aq - a, aq^2 - aq, aq^3 - aq^2, \dots, aq^n - aq^{n-1}$$

$$\underbrace{a(q-1)}_{a_1}, \underbrace{aq(q-1)}_{\text{قد رست}}, \underbrace{aq^2(q-1)}_{\text{قد رست}}, \dots, a(q-1)q^{n-1} \rightarrow \text{قد رست هر دو دنباله مساوی است}$$

$\checkmark q = \text{قد رست}$

$$(a) \quad (a) + (a+d) + (a+2d) + \dots + (a+(n-1)d) = na + \frac{n(n-1)}{2}d = \frac{n}{2}(ra + (n-1)d)$$

$$\Rightarrow S_n = \frac{n}{2}(ra + (n-1)d)$$

$$ب) \quad (a + aq + aq^2 + \dots + aq^{n-1}) \frac{q-1}{q-1}$$

$$= (\cancel{aq} - a + \cancel{aq^2} - \cancel{aq} + \cancel{aq^3} - \cancel{aq^2} + \dots + aq^n - \cancel{aq^{n-1}}) \frac{1}{q-1} = \frac{aq^n - a}{q-1}$$

$$= a \left(\frac{q^n - 1}{q - 1} \right) = S_n \checkmark$$