سینا فاطمی

۱- الف)

$a_n = \frac{1}{2}, 1, 2, \dots$

$a_{10} = a_1 \times q^9 \to \frac{1}{2} \times 2^9 = 2^8 = 256$

ب)

$\frac{a_{17}}{a_{13}} = \frac{a_{13} \times q^4}{a_{13}} = q^4 \qquad 2^4 = 16$

$q = 3 \to \begin{array}{l} a_1 + a_1 q^3 = 28 \\ a + 27a = 28 \to a = 1 \end{array} \qquad 1, 3, 9, 27, \dots$

if $q = \frac{1}{3}$
$a_1 + a_1 q^3 = 28$
$a + \frac{1}{27} a = 28$
$\frac{28}{27} a = 28 \to a = 27$

$27, 9, 3, 1, \dots$

۲۷: بزرگ‌ترین

ج)

a_4 و a_{10} واسطه $= a_7 \to a_7 = a_1 \times q^6 \to \frac{1}{2} \times 2^6 = 2^5 = 32$

د)

$128 = \frac{1}{2} \times q^{n-1} \to 256 = 2^{n-1} \to n = 9$

۲-

$a_5 = 12$
$a_8 = 96 \quad \to q^3 = 8 \to q = 2$
$a_{10} = ?$

$a_{10} = a_5 \times q^5 \to 12 \times 2^5 = 12 \times 32 = 384$

۳- الف)

$\underbrace{a_1 \times \overbrace{a_2 \times a_3 \times a_4}^{a_2 \times a_4} \times a_5}_{a_3 \times a_3} = 243$

$a_3^5 = 243 \to a_3 = 3$

ب)

$a_1 \times a_5 = a_3 \times a_3 = 3 \times 3 = 9$

۴-

$2^a \times 2^b = (4\sqrt{2})^2 \to 2^{a+b} = 32 \to a + b = 5 \to \frac{a+b}{2} = 2/5$

۵-

$a_{11} \times a_3 = a_7^2$

$(1+x)(1-x) = x^2 \to 1 - x^2 = x^2 \to 1 = 2x^2 \to x^2 = \frac{1}{2} \to x = \pm \frac{\sqrt{2}}{2}$

۶-

$a_1 + a_4 = 28 \to a_1 + a_1 q^3 = 28 \to a(1 + q^3) = 28$
$a_2 + a_3 = 12 \quad a_1 q + a_1 q^2 = 12 \quad aq(1+q) = 12 \to \frac{\cancel{a}(q^2 - q + 1)\cancel{(q+1)}}{\cancel{a} q \cancel{(q+1)}} = \frac{28}{12} = \frac{7}{3}$

$\frac{q^2 - q + 1}{q} = \frac{7}{3} \to 3q^2 - 3q + 3 = 7q \to 3q^2 - 10q + 3 = 0$

$q = \frac{+10 \pm \sqrt{100 - 4 \times 9}}{6} = \frac{10 \pm 8}{6} = 3$ یا $\frac{1}{3}$

۷

$aq^{-1} + a + aq = 13 \rightarrow \frac{3}{q} + 3 + 3q = 13 \rightarrow \frac{3 + 3q + 3q^2}{q} = 13$

$3 + 3q + 3q^2 = 13q$

$aq^{-1} \times a \times aq = 27 \rightarrow a^3 = 27 \rightarrow a = 3$

$\times 3$

$3q^2 - 10q + 3 = 0 \rightarrow q^2 - 10q + 9 = 0 \rightarrow \underbrace{(q-9)}_{9}\underbrace{(q-1)}_{1} = 0$

$q = 3$ یا $q = \frac{1}{3}$

$\frac{3}{3}, 3, 3 \times 3 \rightarrow 1, 3, 9$ یا $\frac{3}{\frac{1}{3}}, 3, 3 \times \frac{1}{3} \rightarrow 9, 3, \frac{1}{3}$

۸-

$a_n = 2, 6, 18$

الف) $S_{10} = 2\left(\frac{3^{10} - 1}{3 - 1}\right) = 3^{10} - 1 = 59048$

ب) $P_{10} = \sqrt{(a_1 \times a_{10})^{10}} = \sqrt{(2 \times 2 \times 3^9)^{10}} = \sqrt{2^{20} \times 3^{90}} = 2^{10} \times 3^{45}$

$a_{10} = a_1 \times q^9 \rightarrow 2 \times 3^9$

۹-

$a, aq, aq^2, aq^3, \ldots$

$aq - a, aq^2 - aq, aq^3 - aq^2, \ldots$

$a(q-1), aq(q-1), aq^2(q-1), \ldots$

$\times q \quad \times q$

یک تصاعد هندسی با قدر نسبت q بدست می آید

۱۰-

الف) $S_n = \frac{n}{2}(a_1 + a_n) \rightarrow S_n = \frac{n}{2}(a_1 + a_1 + (n-1) \times d) \rightarrow S_n = \frac{n}{2}(2a_1 + (n-1) \times d)$

$S_n = a\left(\frac{q^n - 1}{q - 1}\right)$

ب) $S_n = a + aq + aq^2 + aq^3 + \ldots + aq^{n-1}$

$q \times S_n = aq + aq^2 + aq^3 + \ldots + aq^n$

$S_n - qS_n = (a - aq) + (aq - aq^2) + (aq^2 - aq^3) + \ldots aq^{n-1} - aq^n$

$S_n - qS_n = a - aq^n \rightarrow S_n(1-q) = a(1-q^n) \rightarrow S_n = \frac{a(1-q^n)}{1-q}$