نام و نام خانوادگی ...طاها جعفری... پاسخنامه تشریحی تکلیف شماره ..۱۰.. کلاس ...دهم...A

۱

$a_n = \frac{1}{2}, 1, 2, \ldots$ ($\times 2$, $\times 2$)

$a_n = a_1 r^{n-1} \Longrightarrow a_{10} = \frac{1}{2} \times 2^{9} = 2^8 = 256$

$128 = 2^7 = a_1 \times r^{n-1} \Longrightarrow 2^7 = \frac{1}{2} \times 2^{n-1} \Longrightarrow 2^8 = 2^{n-1} \Longrightarrow \boxed{n=9}$

$\frac{a_{17}}{a_{13}} = \frac{\cancel{a_1} \times r^{16}}{\cancel{a_1} \times r^{12}} = r^4 = 2^4 = 16$

$2^2 \xleftrightarrow{2^3} 2^5 \xleftrightarrow{2^3} 2^8$

$a_3 = 4 \;/\; a_{10} = 256 \longrightarrow 4, 8, 16, \boxed{32}, 64, 128, 256$

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$a_5 = 12,\; a_8 = 96 \Rightarrow r^3 = \frac{a_1 r^7}{a_1 r^4} = r^3 = 8 = \frac{96}{12} \Rightarrow r = 2 \Rightarrow a_5 = a_1 \times r^4 = 12 \Rightarrow a_1 = \frac{3}{4}$

$a_{10} = a_1 \times r^9 = \frac{3}{\cancel{4}} \times \cancel{2^9}^{\,2^7} = 2^7 \times 3 = \underline{384}$

۳

$\underbrace{a_1 \times a_2 \times a_3 \times a_4 \times a_5}_{a_3^5} = 243 = 3^5 \Longrightarrow a_3^5 = 3^5 \Longrightarrow a_3 = 3$

$a_1 = \frac{1}{3} \;/\; a_2 = 1 \;/\; a_3 = 3 \;/\; a_4 = 9 \;/\; a_5 = 27 \Longrightarrow a_1 \times a_2 \times a_3 \times a_4 \times a_5 = \underline{243}$ ✓

$a_1 \times a_5 = \frac{1}{3} \times 27 = 9$

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$2^a, 4\sqrt{2}, 2^b$ ($\times r$, $\times r$) $\Longrightarrow 2^a \times 2^b = 2^{a+b} = (4\sqrt{2})^2 = 32 = 2^5 \Longrightarrow a+b = 5$

واسطه $= \frac{a+b}{2} =$ ۲/۵ $= \frac{5}{2}$

۵

$a_3 = 1 - x,\; a_7 = x,\; a_{11} = x+1 \Rightarrow x^2 = (x+1)(1-x) = \cancel{x} - x^2 + 1 - \cancel{x} = -x^2 + 1 \Longrightarrow 2x^2 = 1$

$\Longrightarrow x = \pm \frac{1}{\sqrt{2}} = \pm \frac{\sqrt{2}}{2}$

۶

$a_1 + a_4 = 28 \Rightarrow a_1(1+r^3) = 28$

$a_2 + a_3 = 12 \Rightarrow a_1(r+r^2) = 12$

$\Rightarrow \frac{\cancel{a_1}(1+r^3)}{\cancel{a_1}(r+r^2)} = \frac{\cancel{28}^{7}}{\cancel{12}_{3}} \Rightarrow 3 + 3r^3 = 7r + 7r^2 \Rightarrow$

$3r^3 - 7r^2 - 7r + 3 = 0 \Rightarrow$ 1 ✗ 2 ✗ 3 ✓ $\Rightarrow (r-3)(ar^2+br+c) = 0 \Rightarrow ar^3 + br^2 + cr - 3ar^2 - 3br - 3c = 0$

$\Rightarrow \boxed{a=3}$ (ادامه)

$\Rightarrow (b-3a) = -7$

$\Rightarrow (c-3b) = -7$

$\Rightarrow -3c = 3 \Rightarrow \boxed{c=-1}$

$\Rightarrow -3b = -6 \Rightarrow \boxed{b=2}$

$\Rightarrow (r-3)(3r^2+2r-1) = 0$

$r = \frac{-2 \pm \sqrt{4+12}}{6} = \frac{-2 \pm 4}{6} = \boxed{-1}$ ✓, $\boxed{\frac{1}{3}}$ ✓

$r = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$\boxed{3} \Rightarrow 1 + 27 = 28$ ✓ $\Rightarrow a_n = 1, 3, 9, \underline{27}$

$\boxed{-1} \Rightarrow -1 + 1 = 0$ ✗

$\boxed{\frac{1}{3}} \Rightarrow 1 + \frac{1}{27} = \frac{28}{27}$ ✗

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$a_1 + a_2 + a_3 = 13 \Rightarrow a_1(1+r+r^2) = 13 \Rightarrow \frac{3}{r}(1+r+r^2) = 13 \Rightarrow 3 + 3r + 3r^2 = 13r$

$a_1 \times a_2 \times a_3 = 27 \Rightarrow a_1^3 \times r^3 = 27 \Rightarrow a_1 r = 3 \Rightarrow a_1 = \frac{3}{r}$

$3r^2 - 10r + 3 = 0 \Rightarrow r = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{10 \pm \sqrt{100-36}}{6} = \frac{10 \pm 8}{6} = \boxed{3} \to a_1 = \frac{3}{3} = 1$, $\boxed{\frac{1}{3}} \to a_1 = \frac{3}{\frac{1}{3}} = 9$

$\Rightarrow r = 3 \longrightarrow 1, 3, 9$

if $r = \frac{1}{3} \longrightarrow 9, 3, 1$

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$a_n = 2, 6, 18, \ldots$ ($\times 3$, $\times 3$)

$S_n = \frac{a(1-q^n)}{1-q} \Rightarrow S_{10} = \frac{\cancel{2}(1-3^{10})}{\cancel{1-3}_{-2}} = -1 + 59049 = +\underline{59048}$

$P_{10} = a_1 \times a_2 \times a_3 \times \cdots \times a_{10} \Rightarrow P_{10} = a^{10} \times r^{45} = \underline{2^{10} \times 3^{45}}$

$1+2+3+4+5+6+7+8+9 = 45$

۹

$a_1 \xrightarrow{\times r} a_2 \xrightarrow{\times r} a_3 \xrightarrow{\times r} a_4, \ldots$

$(a_1 - a_2), (a_2 - a_3), (a_3 - a_4), \ldots$

$a(1-r), ar(1-r), ar^2(1-r), \ldots$ ($\times r$, $\times r$)

$r = q$

۱۰

$S_n = \frac{n}{2}(2a + (n-1)d)$

$S_n = a + (a+d) + (a+2d) + \cdots + (a+(n-1)d)$

$S_n = (a+(n-1)d) + (a+(n-2)d) + \cdots + a$

(+) $[2a+(n-1)d] \cdots$ ← از آخر جمع کردم :)

$2S_n = n \times [2a+(n-1)d]$ ⇒ n تا از اینها داریم چون از جمع اول و دوم هرکدام باهم برابر

$S_n = \frac{n}{2}[2a+(n-1)d]$

$S_n = a \times \left(\frac{q^n - 1}{q-1}\right) \Rightarrow S_n = a + aq + \cdots + aq^{n-1}$

$qS_n = aq + aq^2 + \cdots + aq^n$

$qS_n - S_n = aq^n - a \Rightarrow S_n(q-1) = a(q^n-1) \Rightarrow S_n = \frac{a(q^n-1)}{q-1}$