

$$a_1 + \frac{a_1 r}{r} = 12 \Rightarrow a_1(1+r) = 12 \Rightarrow \frac{a_1(1+r)}{a_1(r+r^2)} = \frac{12}{12 \times \frac{1}{r}} \Rightarrow r + r^2 = vr + vr^2 \Rightarrow$$

$$\frac{r}{r} - \frac{vr}{r} - \frac{vr^2}{r} + \frac{r^2}{r} = 0 \Rightarrow \textcircled{1} \times \Rightarrow (r-r)(ar^2 + br + c) = 0 \Rightarrow \frac{ar^2 + br + c}{-r^2 - r - r^2} = 0$$

$$\Rightarrow (b-ra) = -v \Rightarrow (c-rb) = -v \Rightarrow -rc = r \Rightarrow c = -1 \Rightarrow -rb = -v \Rightarrow b = r \Rightarrow r = \frac{-r \pm \sqrt{r^2 + 4r}}{2} = \frac{-r \pm r}{2} = \frac{-1}{2}$$

$$a_1 + \frac{a_1 r}{r} + \frac{a_1 r^2}{r} = 12 \Rightarrow a_1(1+r+r^2) = 12 \Rightarrow \frac{1}{r}(1+r+r^2) = 12 \Rightarrow r + r^2 + r^3 = 12r$$

$$a_1 \times \frac{a_1 r}{r} \times \frac{a_1 r^2}{r} = 12 \Rightarrow a_1^3 r^2 = 12 \Rightarrow ar = r \Rightarrow a = \frac{r}{r}$$

$$r^2 - 1 \cdot r + r = 0 \Rightarrow r = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{1 \pm \sqrt{1 - 4}}{2} = \frac{1 \pm 1}{2} = \frac{2}{2} = 1 \rightarrow a_1 = \frac{r}{r} = 1$$

$$a_n = 1, 9, 81, \dots$$

$$S_n = \frac{a(1-q^n)}{1-q} \Rightarrow S_1 = \frac{1(1-1^1)}{1-1} = -1 + 9 \cdot 9 = +9 \cdot 9$$

$$P_1 = a_1 \times \frac{a_1 r}{r} \times \frac{a_1 r^2}{r} \times \dots \times \frac{a_1 r^{n-1}}{r} \Rightarrow P_1 = a_1^n \times r^{n-1} = r^n \times r^{n-1}$$

$$1 + r + r^2 + r^3 + \dots + r^{n-1} = \frac{r^n - 1}{r - 1}$$

$$(a_1 - a_2), (a_2 - a_3), (a_3 - a_4), \dots$$

$$a(1-r), ar(1-r), ar^2(1-r), \dots$$

$$r=4 \checkmark$$

$$S_n = \frac{n}{2} (2a + (n-1)d)$$

$$S_n = a + (a+d) + (a+2d) + \dots + (a+(n-1)d)$$

$$S_n = (a + (n-1)d) + (a + (n-2)d) + \dots + a$$

$$\textcircled{1} [2a + (n-1)d] \dots$$

$$r S_n = n \times [2a + (n-1)d]$$

$$S_n = \frac{n}{2} [2a + (n-1)d]$$

$$S_n = a \times \left(\frac{q^n - 1}{q - 1} \right) \Rightarrow S_n = a + aq + \dots + aq^{n-1}$$

$$q S_n = aq + aq^2 + \dots + aq^n$$

$$q S_n - S_n = aq^n - a \Rightarrow S_n(q-1) = a(q^n - 1)$$

$$a(q^n - 1) \Rightarrow S_n = \frac{a(q^n - 1)}{q - 1}$$