پاسخ تمرین ... دهم ریاضی ، صفحه ۱۴

۱- الف) $a_n = \underbrace{\frac{1}{2}, 1, 2}_{\times 2\ \times 2}, \dots \Rightarrow a_{10} = \frac{1}{2} \times 2^9 = 2^8 = 256$

ب) $\frac{a_{17}}{a_{13}} = q^4 = 2^4 = 16$

ج) $a_n = \sqrt{a \times r^2 \times a \times r^{10}} = \sqrt{a^2 r^{12}} = a r^6 = \underbrace{\frac{1}{2} \times 2^6}_{a_7} = 2^5 = a_7$

د) $128 = \frac{1}{2} \times 2^{n-1} \Rightarrow 256 = 2^{n-1} \Rightarrow n-1 = 8 \Rightarrow \boxed{n = 9}$

۲- $q^3 = \frac{96}{12} = 8 \Rightarrow q = 2 \Rightarrow a_{10} = a_8 q^2 = 96 \times 4 = 384$

۳- الف) $P_5 = (a_3)^5 = 243 \Rightarrow a_3 = 3$

ب) $P_5 = \sqrt{(a_1 \cdot a_5)^5} = 243 \Rightarrow (a_1 \cdot a_5)^5 = 3^{10} \Rightarrow a_1 \cdot a_5 = 3^2 = 9$

۴- $\sqrt{2^a \times 2^b} = 4\sqrt{2} \Rightarrow \sqrt{2^{a+b}} = \sqrt{32} \Rightarrow a + b = 5$

واسطه حسابی $\rightarrow \frac{a+b}{2} = \frac{5}{2} = 2/5$

۵- $(1-x)(1+x) = x^2 \Rightarrow 1 - x^2 = x^2 \Rightarrow 1 = 2x^2 \Rightarrow x^2 = 0/5 \Rightarrow x = \pm\sqrt{\frac{1}{2}}$

$\Rightarrow x = \pm \frac{1}{\sqrt{2}} \Rightarrow x = \pm \frac{\sqrt{2}}{2}$

۶- $\frac{\cancel{a}(q^3+1)}{\cancel{a}(q^2+q)} = \frac{28}{12} \Rightarrow 12q^3 - 28q^2 - 28q + 12 = 0$ (-16, -16) $\Rightarrow$

$$\begin{array}{l|l} 12q^3 - 28q^2 - 28q + 12 & q+1 \\ \hline -12q^3 - 12q^2 & 12q^2 - 40q + 12 \\ \quad -40q^2 - 28q & \\ \quad +40q^2 + 40q & \\ \qquad 12q + 12 & \\ \qquad -12q - 12 & \\ \qquad 0 & \end{array}$$

$\Rightarrow 12q^2 - 40q + 12 = 0 \xRightarrow{\times 12} q^2 - 40q + 144 = 0$

$\Rightarrow (12q - 36)(12q - 4) = 0 \Rightarrow q = 3,\ q = \frac{1}{3}$

$\rightarrow a(q^3+1) = 28 \Rightarrow a(27+1) = 28 \Rightarrow a = 1 \rightarrow 1, 3, 9, 27$

بزرگترین عدد

$\rightarrow a(q^3+1) = 28 \Rightarrow a(\frac{4}{9}) = 28 \Rightarrow a = 27 \rightarrow 27, 9, 3, 1$

۷ـ

$a_1 \times a_2 \times a_3 = ۲۷ \Rightarrow \boxed{a_2 = ۳}$

$\Rightarrow a_1 \times a_3 = ۹ \ , \ a_1 + a_3 = ۱۰ \Rightarrow a_1 = ۱ \ , \ a_3 = ۹$ یا $a_1 = ۹ \ , \ a_3 = ۱$

$a_n = ۱, ۳, ۹, \ldots$ یا $a_n = ۹, ۳, ۱, \ldots$

۸ـ الف)

$$S_{10} = a\left(\frac{q^n - 1}{q-1}\right) = ۲\left(\frac{۳^{۱۰}-1}{\cancel{۲}}\right) = ۳^{۱۰} - ۱ = ۵۹۰۴۸$$

ب)

$$P_{10} = \sqrt{(a_1 . a_{10})^{10}} \rightarrow (a_1 \times a_{10})^5 = (۲ \times ۲ \times ۳^۹)^5 = \boxed{۲^{۱۰} \times ۳^{۴۵}}$$

۹ـ

$$a_n = \frac{a}{q}, a, aq, aq^2, aq^3, \ldots$$

$$\rightarrow a - \frac{a}{q}, \ aq - a, \ aq^2 - aq, \ aq^3 - aq^2$$

$$\rightarrow \frac{a}{q}(q-1), \ a(q-1), \ aq(q-1), \ aq^2(q-1)$$

$\times q \quad \times q \quad \times q \rightarrow$ $\boxed{\text{قدرنسبت } q}$

$$\sqrt{aq(q-1)\frac{a}{q}(q-1)} = \sqrt{a^2(q-1)^2} = a(q-1) \rightarrow \sqrt{a_1 . a_3} = a_2 \rightarrow$$ $\boxed{\text{دنباله هندسی } q}$

۱۰ـ الف)

$$a_n = a, a+d, a+2d, \ldots, a+(n-1)d$$

$$a + a+d + a+2d + \ldots + a(n-1)d \Rightarrow na + (d + 2d + \ldots + (n-1)d)$$

$$\Rightarrow na + (1+2+3+\ldots+n-1)d \rightarrow na + \frac{(n-1+1)(n-1)}{2} = na + n\frac{n-1}{2}d$$

$$\Rightarrow \boxed{\frac{n}{2}(2a + (n-1)d)}$$

ب)

$$S_n = a + aq + aq^2 + \ldots + aq^{n-1}$$

$$q S_n = aq + aq^2 + \ldots + aq^{n-1} + aq^n$$

$$S_n - qS_n = a - aq^n$$

$$\Rightarrow S(1-q) = a(1-q^n) \Rightarrow S = \frac{a(1-q^n)}{(1-q)} \Rightarrow \boxed{S_n = a\left(\frac{1-q^n}{1-q}\right)}$$