

سوال: حدس و آزمایش (فرض و حدس)

$$a_1, a_r, a_c, a_s$$

$$1 \rightarrow r \rightarrow q \rightarrow rV$$

$$a + aq^r = rV \rightarrow a(1+q^r) = \frac{a(1+q)(1+q^r-q)}{aq(1+q)} \cdot \frac{1+q^r-q}{q} = \frac{V}{c}$$

$$aq + aq^r = rV \rightarrow aq(1+q) = \frac{a(1+q)(1+q^r-q)}{aq(1+q)} \cdot \frac{1+q^r-q}{q} = \frac{V}{c}$$

$$aq = cq^r - cq + c \rightarrow cq^r - 1 \cdot q + r = 0 \rightarrow q^r - 1 \cdot q + r = 0 \rightarrow (q - \frac{1}{c})(q - r) = 0$$

$$q = \frac{1}{c} \text{ or } q = r$$

$$a + aq^r = rV \rightarrow a(1+rV) = rV \rightarrow a = 1 \left\{ a\left(\frac{r}{q}\right) = rV \rightarrow a = rV \rightarrow \sqrt{rV} \cdot \sqrt{rV} = rV \right\}$$

$$a_1 + aq + aq^r = rV \rightarrow aq\left(\frac{1}{q} + 1 + q\right)$$

$$a_1 + aq + aq^r = rV \rightarrow a^c q^c = (aq)^r = rV \rightarrow aq = r$$

$$r\left(\frac{1}{q} + 1 + q\right) = rV \rightarrow \frac{r}{q} + r + rq = rV \rightarrow \frac{r}{q} + rq = 1 \rightarrow \frac{r}{q} + rq = 1 \rightarrow q = \left\{r, \frac{1}{c}\right\}$$

$$a_{1..r} = \boxed{1, r, q} \checkmark = \boxed{q, c, 1} \checkmark$$

الف) $S_{1..} = r \left(\frac{r^{10} - 1}{r - 1} \right) = r^{10} - 1 = 29049 - 1 = \boxed{29048} \checkmark$

ب) $a_{10} = r \times q^9 = r \times (r^9) \rightarrow a_1 + a_2 + a_3 + \dots + a_{10} = (r \times r \times r^9)^d$

$$= (r^r \times c^9)^d = \boxed{r^{10} \times r^{9d}} \checkmark$$

$a_n: a, aq, aq^r, aq^c, aq^r, \dots$

$a_1: a(q-1), a(q^r-q'), a(q^c-q^r)$

$q' = \frac{q^r - q^c}{q^r - 1} = \frac{q(q^{r-1} - 1)}{q^r - 1} = q$

$\boxed{q' = q} \checkmark$

$S_n = (a + (a+ad) + (a+2ad) + \dots + (a+(n-1)ad)) \rightarrow rS_n = (ra + (n-1)ad) \cdot n$

$S_n = (a + (n-1)ad) + \dots + (a+ad) + (a+ad) \rightarrow S_n = \frac{n}{r} (rn + (n-1)ad) \checkmark$

$q(a + aq^r + aq^{r^2} + \dots + aq^{r^{n-1}} + aq^{r^n}) = S_n = aq + aq^r + aq^{r^2} + \dots + aq^{r^n} = q \cdot S_n$

$aq^{r^n} + S_n - a = q \cdot S_n \rightarrow aq^{r^n} - a = S_n(q-1) \rightarrow a(q^{r^n} - 1) = S_n(q-1)$

$\rightarrow S_n = a \frac{q^{r^n} - 1}{q - 1} \checkmark \checkmark$