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$$a_n = \frac{1}{r} \times (1, r, r^2, \dots)$$

$$a_n = \frac{1}{r} (r)^{n-1}$$

$$a_1 = \frac{1}{r} (r)^1 = 1 \times r = r \quad \checkmark$$

$$\frac{a_{1r}}{a_{1r}} = \frac{a q^{1r}}{a q^{1r}} = q^r = r^r = 1 \times r = r \quad \checkmark$$

$$a_v = a q^4 = \frac{1}{r} \times r^4 = r^3$$

$$\pm \sqrt{a_n \times a_1} = \pm \sqrt{r \times r^4} = \pm r^2$$

$$\pm r^2 \quad \checkmark$$

$$d_x = \frac{1}{r} (r)^{x-1} = 1 \times r = r^1 = r \Rightarrow (r)^{x-1} = \frac{r^1}{r} = r^0 = 1 \Rightarrow x-1=0 \Rightarrow x=1 \quad \checkmark$$

$$d_\omega = 1 \times r$$

$$d_\lambda = r \times r$$

$$\frac{a_\lambda}{a_\omega} = \frac{a q^r}{a q^1} = q^r = \frac{r^r}{1 \times r} = r \Rightarrow r = r \quad \checkmark$$

$$a_{1r} = a q^r = a q^r q^r \Rightarrow r \times r (r)^r = r^3 \times r^r \quad \checkmark$$

$$d_\lambda = r \times r$$

$$a_1 \times a_r \times a_{r^2} \times a_{r^3} \times a_\omega = a^\omega q^{(1+r+r^2+r^3)} = a^\omega q^{1r} = (a q^r)^\omega = r \times r \times r = r^3 \Rightarrow d_\mu = r^3 \quad \checkmark$$

$$a_1 \times a_\omega = a^r q^r = (a q^r)^r = r^r \quad \checkmark$$

$$r^d \times r^b = (r \sqrt{r})^r \Rightarrow r^{d+b} = r^r = r^r \Rightarrow d+b=r$$

$$\frac{\omega}{r} = r/\omega \quad \checkmark$$

$$d_\mu = 1-x$$

$$a_v = x$$

$$d_{11} = x^{r+1}$$

$$d_\mu d_{11} = d_v \Rightarrow \frac{(1-x)(1+x)}{1-x^r} = x^r \quad \left. \begin{aligned} 1-x^r &= x^r \Rightarrow 1-x^r = 1 \Rightarrow x^r = \frac{1}{r} \\ \Rightarrow x &= \sqrt[r]{\frac{1}{r}} \end{aligned} \right\}$$

مقدار x را بیابید!

$$a_1 + a_r = r\lambda \rightarrow a + aq^r = a(1+q^r) = r\lambda$$

$$a_r + a_{r^2} = r^2 \rightarrow aq + q^2 = a(q+q^r) = r^2$$

$$\frac{a(1+q^r)}{a(q+q^r)} = \frac{r\lambda}{r^2} = \frac{r}{r} \Rightarrow \frac{1+q^r}{q+q^r} = \frac{r}{r} \Rightarrow 1+q^r = q+q^r \Rightarrow q = 1 \Rightarrow a = r\lambda \Rightarrow aq^r = r\lambda \checkmark$$

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$$a_1 + a_r + a_{r^2} = r^3$$

$$a_1 + a_r + a_{r^2} = r^3 = a^3 q^3 = r^3 \Rightarrow (aq)^3 = r^3 \Rightarrow aq = r \Rightarrow \frac{aq}{a} = r$$

$$\left. \begin{array}{l} a_1 + r + a_{r^2} = r^3 \\ \Rightarrow a_1 + a_{r^2} = r^3 - r \\ a_1 r + a_{r^2} = r^3 \\ \Rightarrow a_1 a_{r^2} = r^3 \end{array} \right\}$$

$$\left. \begin{array}{l} a_1 + a_{r^2} = r^3 - r \\ a_1 \times a_{r^2} = r^3 \end{array} \right\} \Rightarrow \begin{array}{l} a_1 = 1, a_{r^2} = r^2 \rightarrow a_n = 1, r, r^2, \dots \\ a_1 = r, a_{r^2} = 1 \rightarrow a_n = r, r^2, r^3, \dots \end{array} \checkmark$$

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$$a_n = r, r^2, r^3, \dots$$

$$\Rightarrow a_n = r(r)^{n-1}$$

$$S_n = a \left(\frac{q^n - 1}{q - 1} \right) \Rightarrow S_n = r \left(\frac{r^n - 1}{r - 1} \right) = r \frac{r^n - 1}{r - 1} = \frac{r^{n+1} - r}{r - 1} \checkmark$$

$$a \times a_r \times a_{r^2} \times \dots \times a_{r^{n-1}} = a \times aq \times aq^2 \times \dots \times aq^{n-1} = a^n \times q^{(1+2+\dots+n-1)} = a^n \times q^{\frac{n(n-1)}{2}} = r^n \times r^{\frac{n(n-1)}{2}} \checkmark$$

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$$a_n = a, a_r, a_{r^2}, \dots, a_{r^{n-1}} \rightarrow a, aq, aq^2, \dots, aq^{n-1}$$

$$a - aq, aq - aq^2, \dots, aq^{n-1} - aq^n$$

$$\Rightarrow a(1-q) + aq(1-q) + \dots + aq^{n-1}(1-q) \checkmark$$

$$\frac{aq(1-q)}{a(1-q)} = q \rightarrow \text{نسبة} \checkmark$$

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$$a_n = a, a_r, a_{r^2}, \dots, a_{r^{n-1}} \rightarrow a, aq, aq^2, \dots, aq^{n-1}$$

$$\Rightarrow a + aq + aq^2 + \dots + aq^{n-1}$$

$$\left\{ \begin{array}{l} ra(n-1)d \\ r + (n-1)d \\ ra(n-1)d \\ \vdots \\ ra(n-1)d \end{array} \right\} \Rightarrow \frac{n}{r} \Rightarrow \frac{n}{r} (ra(n-1)d) \checkmark$$

$$S_n = a + a_r + a_{r^2} + \dots + a_{r^{n-1}} = a + aq + aq^2 + \dots + aq^{n-1}$$

$$\Rightarrow a(1+q+q^2+\dots+q^{n-1}) \xrightarrow{\times q} a(q+q^2+\dots+q^n)$$

$$(q+q^2+\dots+q^n) - (1+q+q^2+\dots+q^{n-1}) = q^n - 1$$

$$q(1+q+q^2+\dots+q^{n-1}) - (1+q+q^2+\dots+q^{n-1}) = q^n - 1$$

$$\Rightarrow a \left(\frac{q^n - 1}{q - 1} \right) \checkmark$$

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