تکلیف شماره ۱۴ کلاس ۸۱۱۵

سوال ۱:

$a_n = \frac{1}{2}, 1, 2, \dots$

الف) $a_n = aq^{n-1} = \frac{1}{2} 2^{n-1}$ $a_{10} = \frac{1}{2} \times 2^9 = 256$

ب) $\frac{a_{17}}{a_{12}}$ $a_{17} = aq^{16}$ $a_{12} = aq^{11}$ $\frac{aq^{16}}{aq^{11}} = q^5 = 2^5 = 16$

ج) $q = \frac{256}{2} = 64$ $\frac{1}{2}2^9, \frac{1}{2}2^2, 256$

د) $a_{10} = 256$ $aq = a_{10} = \frac{256}{2} = 128$ 19

سوال ۲: 192

$a_5 = 12$ $a_1 = a_5 \times q^4$ $12 \times q^4 = 96$ $\frac{96}{12} = q^4 = 8$ $q = 2$

$a_1 = 96$ $a_n = aq^{n-1}$ $12 = q^2$ $16 \times a = 12$ $a = \frac{3}{4}$

$a_{10} = \frac{3}{4} \times 2^9$ $3 \times 128 = 192$

سوال ۳:

$a_1 \times a_2 \times a_3 \times a_4 \times a_5 = 243$

$a_3^5 = 243 \rightarrow a_3 = 3$

الف) 3

سوال ۴:

$2^b, 4\sqrt{2}, 2^a$

$2^b \times 2^a = (4\sqrt{2})^2$ $2^{ab} = 32$ $ab = 5$

$a_2 = 1$ $a = 5$

$b = 5$ $b = 1$

$\frac{a+b}{2} = \frac{6}{2} = 3$

سوال ۵:

a_4, a_7, a_{11}

$1-x, x, x+1$

$a_4 = aq^3$
$a_7 = aq^6$
$a_{11} = aq^{10}$

$x^2 = (1-x)(x+1) = x^2 = x+1-x^2-x$ $x^2 = 1-x^2$

$a_4 = q^4$ $a_7 = q^8$ $a_{11} = q^{12}$

$\frac{aq^6}{aq^3} = q^3 = \frac{x}{1-x}$ $\Rightarrow$ $\frac{aq^{10}}{aq^6} = q^4 = \frac{x+1}{x}$

$\frac{x+1}{x} = \frac{x}{1-x} = x^2 = (1-x)(x+1) = 1-x^2$

$2x^2 = 1$ $x^2 = \pm\frac{1}{2}$ $+\frac{1}{\sqrt{2}}$ $-\frac{1}{\sqrt{2}}$ $\rightarrow$ غیر قابل قبول تعریف غیر حقیقی

سوال ۶:

$a_4 > a_6$

$a + a_6 = 28$

$a_2 + a_3 = 12$

$a + aq^2 = 28$

$aq + aq^2 = 12$

$\frac{a+aq^2}{aq+aq^2} = \frac{28}{12} = \frac{a(1+q^2)}{a(q+q^2)}$

$\frac{1-q+q^2}{q} = \frac{7}{3}$

$\Delta = b^2 - 4ac$

$(-10)^2 = 4 \times 3 \times 3 = 100 - 36 = 64$

$a(1+q^2)\left\{\begin{matrix} \frac{10+8}{4} \\ \frac{10-8}{4} \end{matrix}\right.$

$\frac{(1+q)(1-q+q^2)}{q(1+q)}$

$3q^2 - 10q + 3 = 0$

$q = \frac{-b \pm \sqrt{\Delta}}{2a} = \frac{-(-10) \pm \sqrt{64}}{2 \times 3}$

سوال ۷:

$a_1 \times a_2 \times a_3 = 27$

$a_1 + a_2 + a_3 = 13$

$a_1 + a_3 = 10$

$a_2^2 = 27$

$a_2 = 3$

$a_1 = 1$

$a_3 = 9$

$a_1 = 1$

$a_2 = 3$

$a_3 = 9$

سوال ۸:

$a_n = 3; q, 18$

$S_{10} = a\frac{1-q^n}{1-q}$

$a\frac{q^n - 1}{q-1}$

$S_{10} = a\frac{3^{10}-1}{3-1} = 4 \times \frac{3^{10}-1}{4}$

الف)

$S_{10} = 4 \times 9,049 - 1 = 59,048$

$a_1, a_1 q, a_1 q^2, \dots a_1 q^9$

$\frac{(9+0) \times 10^5}{2} = 45$

ب)

$= a_1^{10} \times q^{45}$

$3^{10} \times 3^{45}$

سوال ۹:

$a, aq, aq^2, aq^3, \dots aq^{n-1}$

$aq - a, aq^2 - aq$

$a(q-1), aq(q-1), aq^2(q-1)$ → جمله عمومی $a_1 q^{n-1}$

$\times q$ $\times q$

$a_1 = a(q-1)$ → $a(q-1)$

سوال ۱۰:

$a_1 + a_2 + a_3 + \dots + a_n = a_1 + a_1 + d + a_1 + 2d + \dots + a_1 + (n-1)d$

مجموع اعداد متوالی $= \frac{(\text{جمله آخر} + \text{جمله اول}) \times \text{تعداد}}{2} = \frac{(a_1 + a_1 + (n-1)d) \times n}{2} = \frac{n}{2}(2a_1 + (n-1)d)$

$S_n = a\left(\frac{q^n - 1}{q-1}\right)$ → $a_1 + a_2 + a_3 + \dots a_n = a_1 + aq + aq^2 + \dots aq^{n-1}$

سوال ۱۰ بخش ب :

$$S_n = a\frac{1-q^n}{1-q} = a+aq+aq^2+\cdots+aq^{n-1} \quad (q\neq 1)$$

$$(S_n = a+aq+aq^2+\cdots+aq^{n-1})\times q$$

$$qS_n = aq+aq^2+aq^3+\cdots+aq^n \qquad S_n - qS_n = (a+aq+\cdots+aq^{n-1}) - (aq+aq^2+\cdots+aq^n)$$

$$S_n(1-q) = a-aq^n \quad q\neq 1 \text{ چون} \rightarrow S_n = \frac{a-aq^n}{1-q} = a\frac{1-q^n}{1-q}$$

$$q=1 \text{ اگر} \rightarrow S_n = a+a+a+\cdots+a = na$$