مهدی موسوی

به نام خدا

۲۵ آفرین!

① (۲)

$a_1 = 40$

$a_4 = a_1 + 3d = 61$ $\Big] \to 3d = 21 \to \boxed{d = 7}$

$a_n = 7n + 33 = 208 \to \boxed{n = 25 \text{ ام}} \to$ جواب

② (۲)

اولین عدد $= 101$

آخرین عدد $= 997$ $\Big] \to 101, 108, 115, \cdots, 997 \to$

$\to$ تعداد $= \frac{997 - 101}{7} + 1 = 129 = n$

③ (۲)

$a_2 + a_3 + a_4 \to a_1 + d + a_1 + 2d + a_1 + 3d \to 3a_1 + 6d = 0$

$a_3 + a_4 + a_5 \to a_1 + 2d + a_1 + 3d + a_1 + 4d \to 3a_1 + 9d = 6$ $\Big] \to 3d = 6$

-36

$d = 2$, $a = -4$ $\Big] \to a_{17} + a_8 = \ldots$

$\to -26 - 8 = -34$

④ (۲)

$a_8 - a_3 = a_{13} - a_8 \to (2^{n+1}) - (2^n - 1) = (2^{2n} - 3) - (2^{n+1}) \Rightarrow 2^n + 1 = 2^{2n} - 3 - 2^{n+1} \Rightarrow$

$2^{2n} - 3 \times 2^n - 4 = 0 \to t^2 - 3t - 4 = 0 \to t = \frac{3 \pm \sqrt{25}}{2} \to t = -1, 4 \xrightarrow{t = 2^n > 0} t = 4$

$a_3 = 3$, $a_8 = 8$, $a_{13} = 13 \to 3 + 8 + 13 = 24$

⑤ (۲)

$a_1 + 9d - (a_1 + 11d) = 5 \to 2d = 5 \to \boxed{d = 2.5}$

$a_1 + 11d + a_1 + 9d = 35 \to 2a_1 + 20d = 25 \to a_1 = -12.5$ $\Big] \to a_{21} = a_1 + 20d = 37.5$

⑥ (۲)

$S_5 = \frac{5}{2}(2a + 4d) = 5(a + 2d)$

$S_{10} = \frac{10}{2}(2a + 9d) = 5(2a + 9d)$ $\Big] \to S_{10} - S_5 = 5(a + 7d)$

شرط $= 5(a + 2d) = \frac{1}{3} \times 5(a + 7d) \to 3a + 6d = a + 7d \to 2a = d$ $\Big] \to \frac{a_2}{a_1} = \frac{a + 2a}{a} = \boxed{3}$

⑦ (۲)

$t_n = a + (n-1)d$, $t_{n-3} = a + (n-4)d \to a + (n-1)d = a + (n-4)d + 15 \to d = 5$

⑧ (۲)

$t_5 = a_1 + 4d = a_1 + 20$

$t_7 = a + 30$

$t_9 = a + 40$ $\Big] \to t_9^2 - t_5^2 = n t_7 \to (a^2 + 80a + 1600) - (a^2 + 40a + 400) = n(a + 30)$

$\Rightarrow 40a + 1200 = n(a + 30) \to \boxed{n = 40}$ جواب

$1200 = 30n \to n = 40$

⑨ (۲)

$d = \frac{b - a}{m + 1} \to \frac{2 + 13\sqrt{3} - 2(1 - 6\sqrt{3})}{13} = \frac{13\sqrt{3}}{13} = \boxed{\sqrt{3} = d}$

⑩ (۲)

$d = \frac{\ldots - 2}{\ldots + 2} = \frac{30}{2(n-1)} = \frac{15}{n-1}$

$a_{n+1} = 11 \to a_{n+1} = a_1 + nd$, $d = \frac{15}{n-1}$

$a_{n+1} = a_1 + \frac{15n}{n-1} = 11 \to \frac{15n}{n-1} = 9 \to \boxed{n = + \ldots}$

$$\left.\begin{array}{l} a_n = a_1 + (n-1)d \\ a_{n+4} = a_1 + (n+3)d \\ a_3 = a_1 + 2d \\ a_{17} = a_1 + 16d \end{array}\right\} \rightarrow a_n + a_{n+4} = a_3 + a_{17} \Rightarrow 2a_1 + (2n+2)d = 2a_1 + 18d \rightarrow$$

$$(2n+2)d = 18d \rightarrow \boxed{n = 8}$$ ✓

(۲)

$$a_n = 2a_{\frac{n}{2}} \longrightarrow a_8 = 2a_4 \rightarrow a_1 + 7d = 2a_1 + 6d \rightarrow a_1 = -d$$

$$d(k-2) = 0 \rightarrow \boxed{k = 2}$$ ✓ جواب