$d = \frac{a_{4} - a_{1}}{4-1} = \frac{61 - 40}{4-1} = \frac{21}{3} = 7$ $\boxed{n = 25}$

$a_n = a_1 + (n-1)d$

$a_n = 40 + (n-1)7 = 7n + 33$ ۱

$7n + 33 = 208$

$7n = 175$

$100 \leq 7n + 3 \leq 999$ تعداد $n = (142 - 14) + 1 = \boxed{129}$

$97 \leq 7n \leq 996$ ۲

$14 \leq n \leq 142$

$a_n = a_1 + (n-1)d \Rightarrow 6 + (n-1)(-2) = 8 - 2n$

$a_{n+1} + a_{n+2} + a_{n+3} = (a_1 + nd) + (a_1 + (n+1)d) + (a_1 + (n+2)d)$

$= 3a_1 + 3nd + 3d = -6n + 12 \xrightarrow{n=2} 3a_1 + 9d = 0$

$\xrightarrow{n=1} 3a_1 + 6d = 6$

$\begin{cases} 3a_1 + 9d = 0 \\ 3a_1 + 6d = 6 \end{cases}$

$3d = -6$

$d = -2 \quad a_1 = 6$

$a_8 = 8 - 16 = -8$

$a_{17} = 8 - 34 = -26$

$\boxed{a_8 + a_{17} = -8 - 26 = -34}$ ۳

$a_8 = \frac{a_3 + a_{13}}{2} \Rightarrow 2a_8 = a_3 + a_{13}$ ← جمله سوم با جمله هشتم و جمله هشتم پنج جمله با جمله ۱۳ فاصله دارد

$2(2^x + 1) = (2^x - 1) + (2^{2x} - 3)$

$= 2^{x+1} + 2 = 2^{2x} + 2^x - 4$

$\Rightarrow 2^x = y$

$4y - y^2 + y - 4 \longrightarrow y^2 - 3y - 4 = 0 \longrightarrow (y-4)(y+1) = 0 \Rightarrow y = 4 = 2^x \quad x = 2$

$a_3 = 3$

$a_8 = 8$ $13 + 8 + 3 = \boxed{24}$ ۴

$a_{13} = 13$

$a_n = a_1 + (n-1)d$

$a_{10} = a_1 + 9d$

$a_{12} = a_1 + 11d$

$a_{10} - a_{12} = a_1 + 9d - a_1 - 11d = -2d = 5 \Rightarrow d = -\frac{5}{2}$ ۵

$a_{10} + a_{12} = 2a_1 + 20d = 25$

$2a_1 + 20\left(-\frac{5}{2}\right) = 25$

$2a_1 = +75 \quad a_1 = 37/5$

$a_{21} = a_1 + 20d = 37/5 - 50 = \boxed{-12/5}$

مجموع ۵ جمله $= \frac{5}{2}(2a+4d)$

$S_5 = 5(a+2d)$

مجموع جملات ۶ تا ۱۰ $= \frac{5}{2}(2(a+5d)+4d)$

$S_{6-10} = 5(a+7d)$

$5(a+2d) = \frac{1}{3}\times 5(a+7d)$

$15(a+2d) = 5(a+7d)$

$3(a+2d) = (a+7d)$

$3a+6d = a+7d$

$2a = d$

$a_2 = a+d = a+2a = 3a$

$a_2 = 3a_1$

جمله دوم ۳ برابر جمله اول است ۶

$t_n - t_{n-3} = 15 = 3d \quad d = 5$

$t_9^2 - t_5^2 = (t_9 - t_5)(t_9 + t_5)$

$t_9 - t_5 = (a+8d)-(a+4d) = 4d = 4\times 5 = 20$ ۷

$t_9 + t_5 = t_7 + t_7$

$t_9^2 - t_5^2 = 20\times 2t_7 = \boxed{40t_7}$

$a_1 = 2(1-6\sqrt{3}) = 2-12\sqrt{3}$

$a_n = 2+\sqrt{3}$

$d = \frac{a_n - a_1}{13} = \frac{2+\sqrt{3}-(2-12\sqrt{3})}{13} = \frac{13\sqrt{3}}{13} = \sqrt{3}$ ۸

تعداد کل جمله ها $= 4n-3+2 = 4n-1$

$a_1 = 2 \qquad a_{4n-1} = 32$

$a_{4n-1} = a_1 + (4n-2)d$

$32 = 2+(4n-2)d \Rightarrow 30 = (4n-2)d$

$d = \frac{15}{2n-1}$

واسطه n ام یعنی جمله n+1 $= a_{n+1} = (a_1)+nd$

$a_{n+1} = 2+nd$

$\left\{ 9 = n\left(\frac{15}{2n-1}\right) \right\} \times 2n-1$ ۹

$9(2n-1) = 15n$

$18n-9 = 15n$

$3n = 9 \qquad \boxed{n=3}$

$a_k = a_1 + (k-1)d$

$a_n + a_{n+4} = a_3 + a_{17}$

$(a_1+(n-1)d) + (a_1+(n+3)d) = a_1+2d+a_1+16d$

$\cancel{2a_1} + 2nd + 2d = \cancel{2a_1} + 18$

$(2 + 2)\cancel{d} = 18\cancel{d} \qquad 2n+2 = 18$

$n = 8$

$a_{n=0} \Rightarrow a_1 + (k-1)d = 0 \Rightarrow a_1 = -d$

$-d + (k-1)d = 0$

$(k-2)d = 0 \xRightarrow{d\neq 0} \boxed{k=2}$

$n=8 \Rightarrow a_n = 3a_{\frac{n}{2}}$ ۱۰

$a_8 = 3a_4$

$a_8 = a_1 + 7d$

$a_4 = a_1 + 3d$

$a_1 + 7d = 3a_1 + 9d$

$a_1 = -d$