

$$\begin{aligned} a_1 &= 40 \\ a_r &= 91 \end{aligned} \quad d = \frac{91-40}{r-1} = 7 \Rightarrow a_n = 7n + 33 \quad (1)$$

$$7n + 33 = 108 \Rightarrow 7n = 175 \Rightarrow n = 25 \quad \text{پس ۲۵ عدد}$$

$$1, 1.8, 1.6, 1.4, \dots$$

$$a_n = 7n + 33 \Rightarrow 7n + 33 \leq 999 \Rightarrow 7n \leq 966 \Rightarrow n = 139 \quad \text{پس ۱۳۹ عدد}$$

$$\underbrace{(a_r + a_r + a_r)}_{-7(2)+12=0} - \underbrace{(a_r + a_r + a_r)}_{-7(1)+12=6} = a_5 - a_r = -6 \Rightarrow d = \frac{-6}{5-2} = -2 \quad (2)$$

$$a_r + a_r + a_r = 6 \Rightarrow (a_r - d) + a_r + (a_r + d) = 6 \quad 3a_r = 6 \Rightarrow a_r = 2$$

$$a_1 - a_r + d = 2 - 10 = -8 \quad a_1 + a_{14} = -8 - 26 = -34$$

$$a_{14} = a_r + 13d = 2 - 26 = -24$$

$$\left. \begin{aligned} a_r &= r^x - 1 \\ a_1 &= r^{x+1} \\ a_{12} &= r^{12} - r \end{aligned} \right\} \Rightarrow r^{12} = r + r^x - 1 = \underbrace{r \times r^{x+1}}_{r^{x+2}} \Rightarrow r^{12} + r^x - r = r^{x+2} \quad \checkmark$$

$$\Rightarrow (r^x)^r + r^x - r^{x+2} - r = 0 \Rightarrow (r^x)^r - r(r^x) - r = 0 \Rightarrow (r^x - r)(r^x + 1) = 0 \quad \text{پس } r^x = r \quad \checkmark$$

$$\begin{aligned} x &= r \rightarrow a_r = r^r - 1 = r \\ a_1 &= r^{r+1} = 1 \\ a_{12} &= r^{12} - r = 12 \end{aligned} \quad a_r + a_1 + a_{12} = r + 1 + 12 = 15$$

$$\left. \begin{aligned} a_1 + a_{12} &= 13 \\ a_{12} - a_1 &= 12 \end{aligned} \right\} \Rightarrow 2a_{12} = 25 \Rightarrow a_{12} = 12.5 \Rightarrow a_1 = 0.5 \Rightarrow d = \frac{12.5 - 0.5}{r} = 2.5 \quad (3)$$

$$a_{11} = a_1 + 10d \Rightarrow a_{11} = 0.5 + 10(2.5) = 25.5$$

$$\frac{r \times d}{x} (ra_1 + fd) = \frac{d}{x} (ra_1 + fd) \Rightarrow ra_1 + fd = ra_1 + fd \quad \checkmark$$

$$\underbrace{a_1 + d}_{a_1} + \underbrace{a_1 + d}_{a_1} \quad fa_1 = rd \Rightarrow ra_1 = d$$

$$\frac{a_r}{a_1} = \frac{a_1 + d}{a_1} = \frac{a_1 + ra_1}{a_1} = r$$

$$t_n = t_{n-r} + 15 \Rightarrow t_7 = t_1 + 15 \xrightarrow{t_7 = t_1 + rd} rd = 15 \Rightarrow d = 15 \quad (4)$$

$$\frac{t_7^r - t_1^r}{t_7} = \frac{(t_1 + t_5)(t_7 - t_1)}{t_7} \Rightarrow \frac{r t_7 (t_7 - t_1)}{t_7} = r(t_7 - t_1) \xrightarrow{t_7 - t_1 = fd} r(fd) = r(r \cdot 0) = 0$$

$$d = \frac{r + \sqrt{r} - r(1 - 4\sqrt{r})}{12+1} = \frac{r + \sqrt{r} - r + 4r\sqrt{r}}{13} = \frac{4r\sqrt{r}}{13} = \sqrt{r} \quad (5)$$

$$\left. \begin{aligned} d &= \frac{r(r-r)}{r(r-r+1)} = \frac{r}{r(r-r)} \\ \text{find } d_1 &= r(n+1) \text{ de} \\ d &= \frac{11-r}{n+1-1} = \frac{r}{n} \end{aligned} \right\} \Rightarrow d = d \Rightarrow \frac{r}{r(r-r)} = \frac{r}{n} \Rightarrow r(r-r+1) = r \cdot n$$

$$\begin{aligned} r(r-r+1) &= r \cdot n \\ r(r-r+1) &= r \cdot n \\ r(r-r+1) &= r \cdot n \\ r(r-r+1) &= r \cdot n \end{aligned}$$

$$a_r + a_{14} = a_n + a_{n+r} \Rightarrow r+14 = n+n+r \Rightarrow n=14$$

$$a_n = r a_{\frac{n}{r}} \xrightarrow{n=14} a_n = r a_r \Rightarrow a_1 + 14d = r(a_1 + rd) \Rightarrow a_1 + 14d = r a_1 + r d$$

$$\Rightarrow r a_1 = -rd \Rightarrow a_1 = -d$$

$$a_1 + (n-1)d = 0 \xrightarrow{a_1 = -d} \underset{\substack{\downarrow \\ a_1}}{-d} + nd - d = 0 \Rightarrow nd = rd \Rightarrow n = r \sim \text{find de}$$