

الف)  $y = x^2 - d \rightarrow -x^2 - y - d \rightarrow x^2 = y + d \rightarrow x = \sqrt{y + d} \rightarrow y + d \geq 0 \rightarrow y \geq -d \rightarrow R_f = [-d, +\infty)$

ب)  $y = x^3 + 1 \rightarrow -x^3 - y + 1 \rightarrow x^3 = y - 1 \rightarrow x = \sqrt[3]{y - 1} \rightarrow R_f = \mathbb{R}$

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الف)  $y = x^2 - 4x + 2 \rightarrow y - 2 = (x - 2)^2 \rightarrow (x - 2) = \sqrt{y - 2} \rightarrow x = 2 + \sqrt{y - 2}$   
 $\Rightarrow y \geq 2 \rightarrow R_f = [2, +\infty)$

ب)  $y = x^2 - 2x + 1 \Rightarrow x^2 - 2x - y + 1 = 0 \rightarrow x = \frac{2 \pm \sqrt{4 - 4(-y+1)}}{2} \Rightarrow x = \frac{2 \pm \sqrt{4y}}{2} \Rightarrow x = 1 \pm \sqrt{y}$   
 $\Rightarrow 1 + \sqrt{y} \geq 0 \rightarrow \sqrt{y} \geq -1 \rightarrow y \geq 0 \rightarrow R_f = [0, +\infty)$

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الف)  $y = \frac{x^2 + 3}{x^2 - 2} \Rightarrow y(x^2 - 2) = x^2 + 3 \Rightarrow yx^2 - 2y = x^2 + 3 \Rightarrow yx^2 - x^2 = 2y + 3 \Rightarrow x^2(y - 1) = 2y + 3 \Rightarrow x^2 = \frac{2y + 3}{y - 1}$   
 $\Rightarrow x = \pm \sqrt{\frac{2y + 3}{y - 1}} \Rightarrow \frac{2y + 3}{y - 1} \geq 0 \Rightarrow \frac{y + 3}{y - 1} \geq 0 \Rightarrow R_f = (-\infty, -\frac{3}{2}] \cup (-1, +\infty)$

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ب)  $y = \frac{x|x| + 1}{|x| - 2} \Rightarrow y|x| - 2y = x|x| + 1 \rightarrow y|x| - x|x| = 2y + 1 \Rightarrow |x|(y - x) = 2y + 1$   
 $\Rightarrow |x| = \frac{2y + 1}{y - x} \rightarrow \frac{1}{x} = \frac{2}{y - x} \Rightarrow R_f = (-\infty, -\frac{1}{2}] \cup (2, +\infty)$

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$y = \frac{1}{x^2 - 4x} \Rightarrow y(x^2 - 4x) = 1 \Rightarrow yx^2 - 4yx - 1 = 0 \rightarrow x = \frac{4y \pm \sqrt{16y^2 + 4y}}{2y} \Rightarrow x = \frac{4y \pm \sqrt{4y(4y + 1)}}{2y}$   
 $\Rightarrow 4y \pm \sqrt{4y(4y + 1)} \geq 0 \Rightarrow 4y(4y + 1) \geq 0 \Rightarrow \frac{1}{4} \geq 0 \Rightarrow y \geq 0 \rightarrow R_f = [0, +\infty)$

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الف)  $y = x^2 - 4x + 2 \cup \text{ext} \left| \begin{array}{l} \frac{-b}{2a} = \frac{4}{2} = 2 \\ 4 \cdot 1 + 2 = 6 \end{array} \right. \rightarrow R_f = [-6, +\infty)$

ب)  $y = -x^2 + 4x + 2 \cap \text{ext} \left| \begin{array}{l} \frac{-b}{2a} = \frac{-4}{-2} = 2 \\ -2 + 16 + 2 = 16 \end{array} \right. \rightarrow R_f = [-\infty, 16]$

الف)  $y = \sqrt{x^2 - 4x + 4} \rightarrow U \text{ ext} \left| \begin{array}{l} \frac{-b}{2a} = \frac{4}{2} = 2 \\ 4 - 16 + 4 = -8 < 0 \end{array} \right. \rightarrow \sqrt{[-2, +\infty)} \rightarrow R_f = [0, +\infty)$

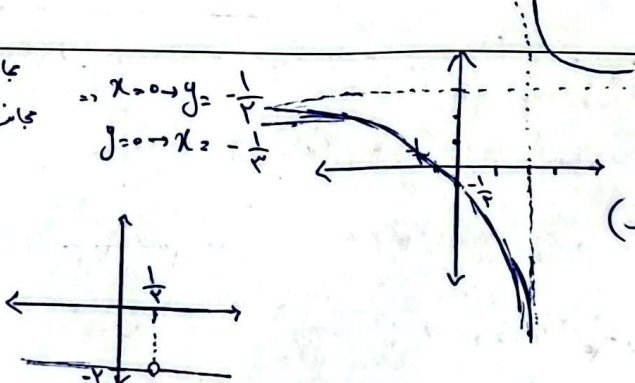
ب)  $y = \sqrt{-x^2 + 4x + 6} \rightarrow \cap \rightarrow \text{ext} \left| \begin{array}{l} \frac{-b}{2a} = \frac{-4}{-2} = 2 \\ -4 + 16 + 6 = 18 > 0 \end{array} \right. \rightarrow \sqrt{(-\infty, 12]} \rightarrow R_f = [0, \sqrt{18}]$

الف)  $y = x^4 + 3x^2 + 2x + 1 \rightarrow R_f = \mathbb{R}$

ب)  $y = \sqrt{x^4 + 4x^2 + 4} \rightarrow R_f = \sqrt{1R} = [0, +\infty)$

الف)  $y = \frac{3x+1}{x-2} \rightarrow R_f = \mathbb{R} - \left\{ \frac{3}{1} \right\} \rightarrow R_f = \mathbb{R} - \{3\}$

ب)  $y = \sqrt{\frac{4x+1}{x+3}} \Rightarrow R_f = \mathbb{R} - \left\{ \frac{4}{1} \right\} \Rightarrow R_f = [0, +\infty) - \{2\}$

الف)  $y = \frac{3x+1}{x-2} \Rightarrow x=2 \rightarrow$  جانب  $x < 2$   $y = 3$  جانب  $x > 2$   $y < 3$   
 $3x-2 \neq 1 \checkmark$   
 ب)  $y = \frac{4x-2}{1-x} \Rightarrow x=1$   
 $4x-2 \neq -2x-2 \checkmark$   


الف)  $y = \cos^2 x + \frac{1}{\cos^2 x} \rightarrow \cos^2 x > 0 \rightarrow R_f = [2, +\infty)$

ب)  $y = \sqrt[3]{\frac{x^2+1}{x}} \Rightarrow \sqrt[3]{\frac{x^2}{x} + \frac{1}{x}} \Rightarrow \sqrt[3]{x + \frac{1}{x}} \Rightarrow \sqrt[3]{(-\infty, -2] \cup [2, +\infty)}$

$\Rightarrow R_f = (-\infty, \sqrt[3]{-2}] \cup [\sqrt[3]{2}, +\infty)$