

Subject:

Year:

Month:

Day:

$$\text{الف) } y = x^2 - d \rightarrow x^2 = y + d \rightarrow x = \pm \sqrt{d+y} \rightarrow y + d \geq 0 \rightarrow y \geq -d \rightarrow R_f = [-d, +\infty) \quad (1)$$

$$\text{ب) } y = x^3 + 1 \rightarrow x^3 = y - 1 \rightarrow x = \sqrt[3]{y-1} \rightarrow R_f = \mathbb{R}$$

$$\text{الف) } y = x^2 - \frac{r}{x} \rightarrow y - r = (x - \frac{r}{x})^2 \rightarrow \pm \sqrt{y-r} = x - \frac{r}{x} \rightarrow \pm \sqrt{y-r} + \frac{r}{x} = x \rightarrow y - r \geq 0 \rightarrow y \geq r \rightarrow R_f = [r, +\infty)$$

$$\text{ب) } y = x^2 - dx + 1 \rightarrow x^2 - dx - y + 1 = 0 \rightarrow x = \frac{d \pm \sqrt{d^2 + 4y}}{2} \rightarrow d + \sqrt{d^2 + 4y} \geq 0 \rightarrow \sqrt{d^2 + 4y} \geq -d \rightarrow y \geq -\frac{d^2}{4} \rightarrow R_f = [-\frac{d^2}{4}, +\infty)$$

$$\text{الف) } y = \frac{x^2 + r}{x^2 - r} \rightarrow yx^2 - ry = x^2 + r \rightarrow yx^2 - x^2 = ry + r \rightarrow x^2(y-1) = r(y+1) \rightarrow x^2 = \frac{r(y+1)}{y-1} \rightarrow x = \pm \sqrt{\frac{r(y+1)}{y-1}} \rightarrow \frac{r(y+1)}{y-1} \geq 0 \rightarrow \frac{-\frac{r}{y} + 1}{+\frac{1}{y} - \frac{1}{y} +} R = (-\infty, -\frac{r}{r}) \cup (1, +\infty)$$

$$\text{ب) } y = \frac{rx + 1}{|x| - r} \rightarrow y|x| - ry = rx + 1 \rightarrow y|x| - r|x| = ry + 1 \rightarrow |x|(y-r) = ry + 1 \rightarrow |x| = \frac{ry+1}{y-r} \rightarrow \frac{-\frac{1}{y} + \frac{r}{y}}{+\frac{1}{y} - \frac{1}{y} +} \rightarrow R = (-\infty, -\frac{1}{r}) \cup (r, +\infty)$$

$$y = \frac{1}{x^2 - rx} \rightarrow yx^2 - rxy - 1 = 0 \rightarrow x = \frac{ry \pm \sqrt{r^2y^2 + 4y}}{2y} \rightarrow r^2y^2 + 4y \geq 0 \rightarrow ry(ry+1) \geq 0 \rightarrow \frac{-\frac{1}{y} + \frac{r}{y}}{+\frac{1}{y} - \frac{1}{y} +} ry \neq 0 \rightarrow y \neq 0 \rightarrow R_f = (-\infty, -\frac{1}{r}) \cup (0, +\infty)$$

$$\text{الف) } y = x^2 - rx + r \quad \cup \quad \text{ext} \begin{cases} -\frac{b}{r_a} = \frac{r}{r} = r \\ 9 - 12 + r = -r \end{cases} \rightarrow R_f = [-r, +\infty) \quad (2)$$

$$\text{ب) } y = -x^2 + rx + r \quad \cap \quad \text{ext} \begin{cases} -\frac{b}{r_a} = \frac{r}{-r} = -r \\ -r + 1 + r = r \end{cases} \rightarrow R_f = (-\infty, r]$$

$$\text{الف) } y = \sqrt{x^2 - rx + r} \rightarrow \cup \rightarrow \text{ext} \begin{cases} \frac{r}{r} = r \\ 9 - 12 + r = -r \end{cases} \Rightarrow \sqrt{[-r, +\infty)} \Rightarrow R_f = [0, +\infty) \quad (3)$$

$$\text{ب) } y = \sqrt{-x^2 + rx + r} \rightarrow \cap \rightarrow \text{ext} \begin{cases} \frac{r}{-r} = -r \\ -r + 1 + r = r \end{cases} \Rightarrow \sqrt{(-\infty, r]} \Rightarrow R_f = [0, \sqrt{1+r}]$$

$$\text{الف) } y = x^4 + rx^2 + rx + 1 \rightarrow R_f = \mathbb{R} \quad (4)$$

$$\text{ب) } y = \sqrt{x^4 + rx^2 + rx + 1} \rightarrow R_f = \sqrt{\mathbb{R}} = [0, +\infty)$$

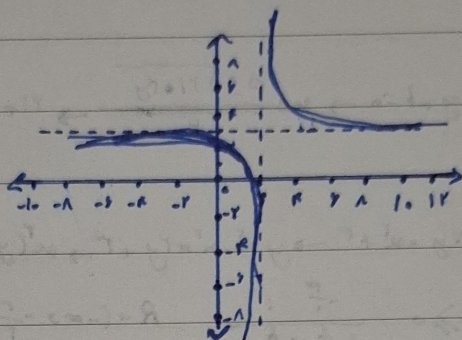
الف) $y = \frac{r^2x+1}{x-r} \rightarrow \frac{a}{c} = \frac{r}{1} = r \rightarrow R_f = \mathbb{R} - \{r\}$

ب) $y = \sqrt{\frac{r^2x+1}{x-r}} \rightarrow \frac{a}{c} = \frac{r}{1} = r \rightarrow R_f = [0, +\infty) - \{r\}$

الف) $y = \frac{r^2x+1}{x-r}$

$y = 0 \rightarrow r^2x+1 = 0 \rightarrow x = -\frac{1}{r^2}$

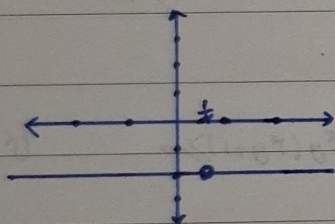
$x = 0 \rightarrow y = \frac{1}{-r} = -\frac{1}{r}$



جانب قائم

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ج) $y = \frac{rx-r}{1-rx} = \frac{r(x-1)}{-rx+1} \rightarrow R_f = \mathbb{R} - \{1/r\} \rightarrow \frac{rx-r}{-rx+1} = \frac{-r(-x+1)}{-rx+1} = -r = y$
 $1-rx \neq 0 \rightarrow x \neq \frac{1}{r}$



الف) $y = \cos^2 x + \frac{1}{\cos^2 x} \rightarrow \cos^2 x \geq 0 \rightarrow R_f = [1, +\infty)$

(1.)

ب) $y = \sqrt{\frac{x^2+1}{x}} = \sqrt{\frac{x^2}{x} + \frac{1}{x}} = \sqrt{x + \frac{1}{x}} = \sqrt{(-\infty, -r] \cup [r, +\infty)} \rightarrow R_f = (-\infty, -\sqrt{r}] \cup [\sqrt{r}, +\infty)$