

$$\text{الف) } y = x^2 - a \rightarrow x^2 = y + a \rightarrow x = \pm \sqrt{y+a} \quad y + a \geq 0 \quad (1)$$

$$y \geq -a \quad R_f = [-a, +\infty)$$

$$\text{ب) } y = x^2 + 1 \rightarrow x^2 = y - 1 \rightarrow x = \pm \sqrt{y-1} \rightarrow R_f = \mathbb{R}$$

$$\text{الف) } y = x^2 - 4x + 4 \rightarrow y = (x-2)^2 \rightarrow x = \pm \sqrt{y} + 2 \quad (2)$$

$$R_f = [0, +\infty)$$

$$\text{ب) } y = x^2 - 2ax + a^2 \rightarrow \underbrace{1}_{a} x^2 - \underbrace{2a}_{b} x + \underbrace{a^2}_{c} - y = 0 \rightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\rightarrow 2a - 4(1-y) \geq 0 \rightarrow 2a - 4 + 4y \geq 0 \rightarrow y \geq \frac{-2a + 4}{4} \rightarrow R_f = \left[\frac{-2a + 4}{4}, +\infty \right)$$

$$\text{الف) } y = \frac{x^2 + 2}{x^2 - 2} \rightarrow x^2 = \frac{y + 2}{y - 2} \rightarrow x = \pm \sqrt{\frac{y + 2}{y - 2}} \quad (3)$$

$$\rightarrow \frac{-\frac{2}{y-2}}{+\infty - \frac{1}{y-2}} \rightarrow R_f = (1, +\infty) \cup (-\infty, -\frac{2}{y-2}]$$

$$\text{ب) } y = \frac{2|x| + 1}{|x| - 2} \rightarrow |x| = \frac{2y + 1}{y - 2} \rightarrow \frac{-\frac{1}{y-2}}{+\infty - \frac{1}{y-2}} \rightarrow R_f = (-\infty, -\frac{1}{y-2}] \cup (2, +\infty)$$

الف) $y = \frac{x^2 + 1}{x^2 - 2} \rightarrow y - 1 = \frac{3}{x^2 - 2}$
 $\rightarrow \frac{-\frac{3}{x^2 - 2}}{1 + \frac{3}{x^2 - 2}} \rightarrow R_f = (-1, +\infty) \cup (-\infty, -\frac{2}{3}]$

ب) $y = \frac{2|x| + 1}{|x| - 2} \rightarrow |x| = \frac{\epsilon y + 1}{y - 2} \rightarrow \frac{-\frac{1}{\epsilon} + 2}{1 + \frac{1}{y - 2}} \rightarrow R_f = (-\infty, -\frac{1}{\epsilon}] \cup (2, +\infty)$

$y = \frac{1}{x^2 - \epsilon x} \rightarrow yx^2 - \epsilon yx - 1 = 0 \rightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
 $14) y^2 + \epsilon a \geq 0 \rightarrow \epsilon y(\epsilon y + 4) \geq 0$
 $2a \neq 0 \rightarrow y \neq 0$
 $\rightarrow R_f = (-\infty, -\frac{1}{\epsilon}] \cup (0, +\infty)$

ج) $y = \frac{1}{x^2 - 9} \rightarrow \begin{cases} x_{min} = 3 \\ y_{min} = -\frac{1}{9} \end{cases} \rightarrow R_f = [-\frac{1}{9}, +\infty)$
 ب) $y = -x^2 + 4x + 2 \rightarrow \begin{cases} x_{max} = 2 \\ y_{max} = 6 \end{cases} \rightarrow R_f = (-\infty, 6]$

د) $y = \sqrt{x^2 - 4x + 2} \rightarrow \begin{cases} x_{min} = 2 \\ y_{min} = 0 \end{cases} \rightarrow R_f = [0, +\infty)$

Hashtag

H A S H T A G

Subject:

Date: / /

$$\Rightarrow y = \sqrt{-x^2 + 6x + 6} \xrightarrow{a < 0} \begin{cases} x_{\max} = 3 \\ y_{\max} = 1\sqrt{6} \rightarrow [0, \sqrt{16}] \end{cases}$$

$$\text{مثال 1 } y = x^2 + 6x + 1 \rightarrow R_f = \mathbb{R}$$

② - V

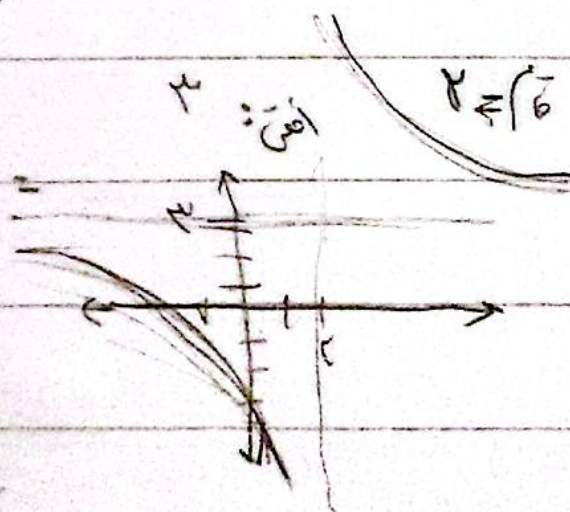
$$\Rightarrow \sqrt{x^2 + 6x + 1} \rightarrow [0, +\infty)$$

$$\text{مثال 2 } y = \frac{6x+1}{x-2} \rightarrow R_f = \mathbb{R} - \left\{ \frac{1}{-1} \right\} \Rightarrow \sqrt{\frac{6x+1}{x-2}}$$

② - 1

$$\hookrightarrow R_f = [0, +\infty) - \left\{ \frac{1}{-1} \right\}$$

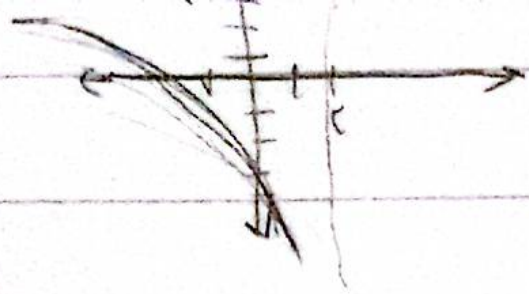
$$\text{مثال 3 } y = \frac{6x+1}{x-2}$$



② - 9

$$\Rightarrow y = \frac{6x-1}{x-2}$$

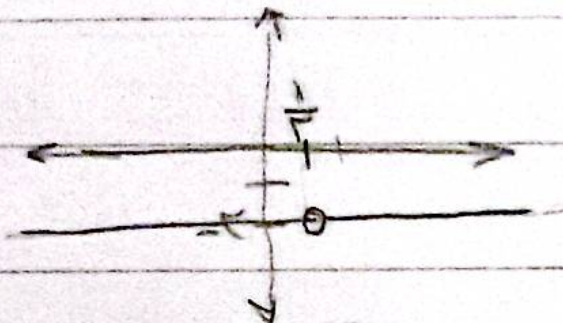
$$= \frac{-1(1-6x)}{x-2} = -1$$



$$\Rightarrow y = \frac{rx - r}{1 - rx}$$

$$x \neq \frac{1}{r}$$

$$= \frac{-r(1 - rx)}{1 - rx} = -r$$



$$\text{ii) } y = \cos^r x + \frac{1}{\cos^r x} \rightarrow \cos^r x \geq 0 \rightarrow R_f = [r, +\infty)$$

$$\Rightarrow y = \sqrt[r]{x} + \frac{1}{\sqrt[r]{x}} \rightarrow \begin{cases} x > 0, & y \geq \sqrt[r]{r} \\ x < 0, & y \leq -\sqrt[r]{r} \end{cases}$$

$$\rightarrow R_f = (-\infty, -\sqrt[r]{r}] \cup [\sqrt[r]{r}, +\infty)$$

②