سوال ۱)

الف) $x^2 = y+5 \rightarrow x = \pm\sqrt{y+5} \rightarrow y+5 \geq 0 \rightarrow y \geq -5 \Rightarrow R_f = [-5, +\infty)$

ب) $x^3 = y-1 \rightarrow x = \sqrt[3]{y-1} \rightarrow R_f = \mathbb{R}$ (فرجه فرد)

سوال ۲)

الف) $x^2 - 4x + 2 - y = 0 \rightarrow x = \frac{-b \pm \sqrt{b^2-4ac}}{2a} \rightarrow x = \frac{4 \pm \sqrt{(-4)^2-4(1)(2-y)}}{2} \Rightarrow$ زیر رادیکال فرجه زوج ≥ ۰

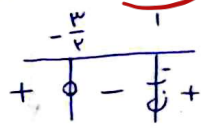
$16 - 24 + 4y \geq 0 \rightarrow 4y \geq 8 \rightarrow y \geq 2 \Rightarrow R_f = [2, +\infty)$

ب) $x^2 - 5x + 1 - y = 0 \rightarrow x = \frac{-b \pm \sqrt{b^2-4ac}}{2a} \rightarrow x = \frac{5 \pm \sqrt{(-5)^2-4(1)(1-y)}}{2}$ زیر رادیکال فرجه زوج ≥ ۰

$25 - 4 + 4y \geq 0 \rightarrow 4y \geq -21 \rightarrow y \geq \frac{-21}{4} \Rightarrow R_f = [\frac{-21}{4}, +\infty)$

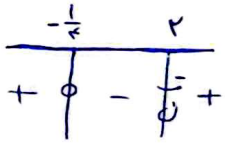
سوال ۳)

الف) $y = \frac{x^2+3}{x^2-2} \rightarrow yx^2 - 2y = x^2 + 3 \rightarrow yx^2 - x^2 = 3 + 2y \rightarrow x^2 = \frac{3+2y}{y-1} \rightarrow x = \pm\sqrt{\frac{3+2y}{y-1}}$



$R_f = (-\infty, -\frac{3}{2}] \cup (1, +\infty) \Leftarrow$

ب) $y = \frac{2|x|+1}{|x|-4} \rightarrow y|x| - 4y = 2|x| + 1 \rightarrow |x|(y-2) = 1+4y \rightarrow |x| = \frac{1+4y}{y-2}$



$R_f = (-\infty, -\frac{1}{4}] \cup (2, +\infty) \leftarrow$

سوال ۴)

$y = \frac{1}{x^2-4x} \rightarrow yx^2 - 4yx - 1 = 0$

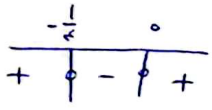
$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$x = \frac{4y \pm \sqrt{(-4y)^2-4(y)(-1)}}{2(y)} = \frac{4y \pm \sqrt{16y^2+4y}}{2y}$

زیر رادیکال فرجه زوج $\geq 0 \rightarrow$

$16y^2 + 4y \geq 0$

$4y(4y+1) \geq 0$



مخرج $\neq 0$

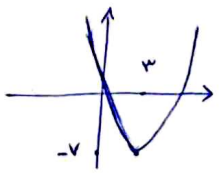
$2y \neq 0$

$y \neq 0$

$R_f = (-\infty, -\frac{1}{4}] \cup (0, +\infty)$

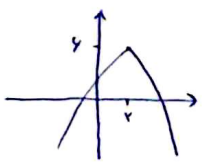
سوال ۵)

الف) $Min \begin{cases} -\frac{b}{2a} = \frac{4}{2} = 3 \\ 3^2 - 6(3) + 2 = -7 \end{cases}$



$R_f = [-7, +\infty)$

ب) $Max \begin{cases} \frac{-b}{2a} = \frac{-4}{-2} = 2 \\ -2^2 + 4(2) + 2 = 6 \end{cases}$

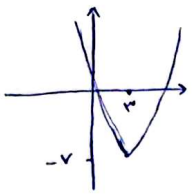


$R_f = (-\infty, 6]$

CamScanner

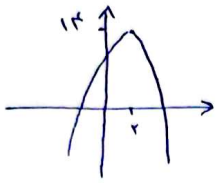
سوال ۶)

الف) $Min \begin{cases} \frac{-b}{2a} = \frac{6}{2} = 3 \\ 3^2 - 6(3) + 2 = -7 \end{cases}$



$Rf = \sqrt{[-7, +\infty)} \Rightarrow Rf = [0, +\infty)$

ب) $Max \begin{cases} \frac{-b}{2a} = \frac{-4}{-2} = 2 \\ -2^2 + 4(2) + 10 = 14 \end{cases}$



$Rf = \sqrt{(-\infty, 14]} \longrightarrow Rf = [0, \sqrt{14}]$

سوال ۷)

الف) $Rf = \mathbb{R}$

ب) $Rf = \sqrt{\mathbb{R}} \longrightarrow Rf = [0, +\infty)$

سوال ۸)

الف) $Rf = \mathbb{R} - \{\frac{3}{1}\} = \mathbb{R} - \{3\}$

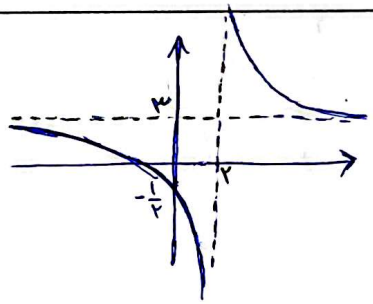
$\frac{a}{c}$

ب) $Rf = \sqrt{\mathbb{R} - \{\frac{4}{1}\}} \longrightarrow Rf = [0, +\infty) - \{2\}$

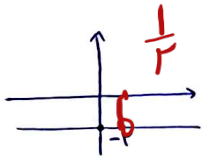
سوال ۹)

الف) $\begin{cases} 2 \Rightarrow \text{ریشه‌ی مخرج} = \text{مجانب قائم} \\ \frac{a}{c} \Rightarrow 3 = \text{مجانب افقی} \end{cases}$

به ازای صفر $\leftarrow -\frac{1}{2}$



ب) $y = \frac{4x - 2}{1 - 2x} = \frac{-2(-2x+1)}{(-2x+1)} = -2$



$1 - 2x \neq 0 \rightarrow x \neq \frac{1}{2}$

سوال ۱۰)

الف) $y = \cos^2 x + \frac{1}{\cos^2 x} \longrightarrow a + \frac{1}{a}, \quad a > 0 \Longrightarrow Rf = [2, +\infty)$

ب) $y = \sqrt[3]{\frac{x^2}{x} + \frac{1}{x}} = \sqrt[3]{\underbrace{x + \frac{1}{x}}_{a + \frac{1}{a}}} \longrightarrow \sqrt[3]{(-\infty, -2] \cup [2, +\infty)} \Rightarrow Rf = (-\infty, \sqrt[3]{-2}] \cup [\sqrt[3]{2}, +\infty)$