۱- الف) $y = x^2 - 5 \rightarrow y + 5 = x^2 \Rightarrow x = \pm\sqrt{y+5}$

$y + 5 \geq 0$
$y \geq -5 \Rightarrow y = [-5, +\infty)$

ب) $y = x^3 + 1 \rightarrow y - 1 = x^3 \Rightarrow x = \sqrt[3]{y-1} \Rightarrow y \in \mathbb{R}$

۲- الف) $y = x^2 - 4x + 6 \rightarrow y = (x-2)^2 + 2 \Rightarrow y - 2 = (x-2)^2$

$(x-2) = \pm\sqrt{y-2} \Rightarrow x = \pm\sqrt{y-2} + 2 \Rightarrow y - 2 \geq 0$
$y \geq 2 \Rightarrow R_f = [2, +\infty)$

ب) $y = x^2 - 5x + 1 \rightarrow y = (x - \frac{5}{2})^2 - \frac{21}{4} \rightarrow y + \frac{21}{4} = (x - \frac{5}{2})^2$

$x - \frac{5}{2} = \pm\sqrt{y + \frac{21}{4}} \rightarrow x = \pm\sqrt{y + \frac{21}{4}} + \frac{5}{2} \Rightarrow y + \frac{21}{4} \geq 0$
$y \geq -\frac{21}{4} \Rightarrow R_f = [-\frac{21}{4}, +\infty)$

۳- الف) $y = \frac{x^2+3}{x^2-2} \Rightarrow yx^2 - 2y = x^2 + 3 \rightarrow yx^2 - x^2 - 2y - 3 = 0$

$x^2(y-1) - 2y - 3 = 0 \rightarrow x^2(y-1) = 2y + 3 \rightarrow x^2 = \frac{2y+3}{y-1} \Rightarrow$

$x = \pm\sqrt{\frac{2y+3}{y-1}} \geq 0$; $\quad -\frac{3}{2}$, 1 : $+ \mid - \nmid +$ $\Rightarrow R_f = (-\infty, -\frac{3}{2}] \cup (1, +\infty)$

$1 \quad -\frac{3}{2}$

ب) $y = \frac{2|x| + 1}{|x| - 4} \Rightarrow y|x| - 4y = 2|x| + 1 \Rightarrow y|x| - 2|x| = 4y + 1$

$\Rightarrow |x|(y-2) = 4y + 1 \rightarrow |x| = \frac{4y+1}{y-2} \Rightarrow$ $\quad -\frac{1}{4}$, 2 : $+ \mid - \nmid +$

مثبت و صفر

$R_f = (-\infty, -\frac{1}{4}] \cup (2, +\infty)$

$a=y \quad b=-4y \quad c=-1$

۴ـ $y=\dfrac{1}{x^2-4x} \rightarrow yx^2-4xy=1 \rightarrow \underbrace{y}_{a}x^2\underbrace{-4}_{b}xy\underbrace{-1}_{c}=0$

$\Delta=(-4y)^2-(4\times -1\times y)=(-4y)^2+4y$

$x=\dfrac{-(-4y)\pm\sqrt{(-4y)^2+4y}}{2y} \rightarrow 16y^2+4y\geqslant 0 \rightarrow y(16y+4)\geqslant 0$

$y\neq 0 \qquad -\dfrac{4}{16}=-\dfrac{1}{4}$

$+ \quad -\tfrac{1}{4} \quad - \quad 0 \quad +$

$R_f=(-\infty,-\tfrac{1}{4}]\cup(0,+\infty)$

۵ـ الف) $y=x^2-6x+2$

$a>0 \rightarrow \min \Rightarrow R_f=[-7,+\infty)$

$y_{min}=\dfrac{-\Delta}{4a}=\dfrac{-(b^2-4ac)}{4a}=\dfrac{-(36-(4\times 2\times 1))}{4}=\dfrac{-28}{4}=-7$

ب) $y=-x^2+4x+2$

$a<0 \rightarrow \max \Rightarrow R_f(-\infty,6]$

$y_{max}=\dfrac{-\Delta}{4a}=\dfrac{-(b^2-4ac)}{4a}=\dfrac{-(16-(4\times 2\times -1))}{-4}=\dfrac{-24}{-4}=+6$

۶ـ الف) $y=\sqrt{x^2-6x+2}$

$a>0 \rightarrow \min \Rightarrow R_f=[-7,+\infty) \Rightarrow \sqrt{[-7,+\infty)} \Rightarrow R_f\,[0,+\infty)$

$y_{min}=\dfrac{-\Delta}{4a}=\dfrac{-(36-(4\times 2\times 1))}{4}=-7$

ب) $y=\sqrt{-x^2+4x+10} \Rightarrow$

$a<0 \rightarrow \max = R_f = \sqrt{(-\infty,14]} \Rightarrow [0,\sqrt{14}]$

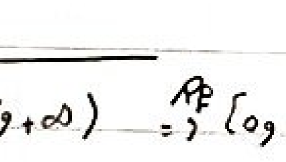
$y_{max}=\dfrac{-\Delta}{4a}=\dfrac{-(16-(4\times 10\times -1))}{-4}=\dfrac{56}{4}=14$

۷ـ بزرگترین توان آن فرد است

الف) $y=x^5+3x^4+2x+1 \Rightarrow R_f=\mathbb{R}$

ب) $y=\sqrt{x^5+4x^2+6x+1} \Rightarrow \sqrt{\mathbb{R}}=[0,+\infty)$

۸ـ الف) $y=\dfrac{3x+1}{x-2} \quad R_f=\mathbb{R}-\{3\}$



$\dfrac{a}{c}=\dfrac{3}{1}=3=$ مجانب افقی $\rightarrow x=0 \Rightarrow \dfrac{1}{-2}$

مجانب قائم $= x-2=0 \rightarrow x=2$

ب) $y=\sqrt{\frac{4x+1}{x+3}} \to \sqrt{R-\{-3\}} \Rightarrow [0,2)\cup(2,+\infty)$

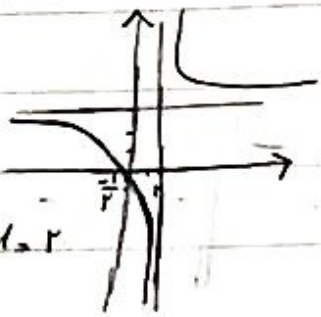
$R-\{-3\}$

$x\to 0 \to \frac{1}{3}$

الف) $y=\frac{3x+1}{x-2} \Rightarrow$ مجانب افقی $=\frac{a}{c}=3$

مجانب عمودی $= x-2=0 \to x=2$

$x\to 0 \to y=\frac{-1}{2}$



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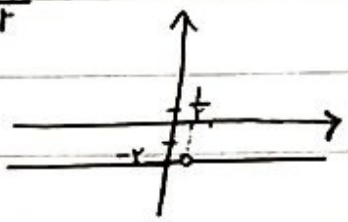
ب) $\frac{4x-2}{1-2x} \Rightarrow \frac{2(2x-1)}{1-2x}=-2$

$1-2x=0$

$1=2x$

$x\neq\frac{1}{2}$



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الف) $y=\cos^2 x+\frac{1}{\cos^2 x} \Rightarrow \cos^2 x>0 \Rightarrow Rf=[2,+\infty)$

ب) $y=\sqrt[3]{\frac{x^2+1}{x}} \Rightarrow \sqrt[3]{\underbrace{x+\frac{1}{x}}_{\in R}} \Rightarrow \sqrt[3]{(-\infty,-2]\cup[2,+\infty)} \Rightarrow$

$Rf=(-\infty,\sqrt[3]{-2}]\cup[\sqrt[3]{2},+\infty)$