

الف) $y = x^2 - 8$ $x^2 = y + 8$ $x = \pm \sqrt{y+8}$ $y+8 \geq 0$ $y \geq -8$ $R_f = [-8, \infty)$

ب) $y = x^2 + 1$ $x^2 = y - 1$ $x = \pm \sqrt{y-1}$ $R_f = \mathbb{R}$

الف) $y = x^2 - 4x + 4$ $x^2 - 4x + 4 - y = 0$ $x = \frac{4 \pm \sqrt{16 - 4(4-y)}}{2}$
 $16 - 4^2 + 4y \geq 0$ $4y - 16 \geq 0$ $4y \geq 16$ $y \geq 4$ $R_f = [4, \infty)$

ب) $y = x^2 - 8x + 1$ $x^2 - 8x + 1 - y = 0$ $x = \frac{8 \pm \sqrt{64 - 4(1-y)}}{2}$
 $64 - 4^2 + 4y \geq 0$ $4y \geq -48$ $y \geq -12$ $R_f = [-12, \infty)$

الف) $y = \frac{x^2+3}{x^2-1} = y$ $x^2 = -\frac{-2y-3}{y-1} = \frac{2y+3}{y-1}$ $x = \pm \sqrt{\frac{2y+3}{y-1}}$ $\frac{2y+3}{y-1} \geq 0$
 $R_f = (-\infty, \frac{3}{2}] \cup (1, \infty)$

ب) $y = \frac{1}{|x|-1} = y$ $|x| = -\frac{-y-1}{y-1} = \frac{y+1}{y-1}$ $\frac{y+1}{y-1} \geq 0$
 $R_f = (-\infty, \frac{1}{2}] \cup (1, \infty)$

$y = \frac{1}{x^2 - 4x}$ $yx^2 - 4yx = 1$ $yx^2 - 4yx - 1 = 0$
 $x = \frac{4y \pm \sqrt{16y^2 + 4y}}{2y}$ $16y^2 + 4y \geq 0$ $4y(4y+1) \geq 0$
 $4y \neq 0 \rightarrow y \neq 0$
 $R_f = (-\infty, \frac{1}{4}] \cup (0, \infty)$

الف) $y = x^2 - 4x + 2 \xrightarrow{a>0} \text{Min} \left| \frac{-b}{2a} = \frac{4}{2} = 2 \right.$ $R_f = [-2, \infty)$
 $x^2 - 4x + 2 = -2$

ب) $y = -x^2 + 4x + 2 \xrightarrow{a<0} \text{Max} \left| \frac{-b}{-2a} = 2 \right.$ $R_f = (-\infty, 4]$
 $-x^2 + 4x + 2 = 4$

الف) $y = \sqrt{x^2 - 4x + 4}$ $x^2 - 4x + 4 \xrightarrow{a > 0} \text{Min} \left| \frac{4}{4} = 1 \right.$
 $9 - 11 + 4 = 2$

$R_f = \sqrt{[-1, \infty)} = [0, \infty)$

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ب) $y = \sqrt{-x^2 + 12x + 10}$ $-x^2 + 12x + 10 \xrightarrow{a < 0} \text{Max} \left| \frac{-12}{-2} = 6 \right.$
 $-12 + 11 + 10 = 9$

$R_f = \sqrt{(-\infty, 9]} = [0, \sqrt{9}]$

الف) $y = x^2 + 3x^2 + 2x + 1 \rightarrow R_f = \mathbb{R}$

ب) $y = \sqrt{x^2 + 3x^2 + 4x + 1} \rightarrow R_f = \sqrt{\mathbb{R}} = [0, \infty)$

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الف) $y = \frac{3x+1}{x-2} \rightarrow R_f = \mathbb{R} - \{3\}$

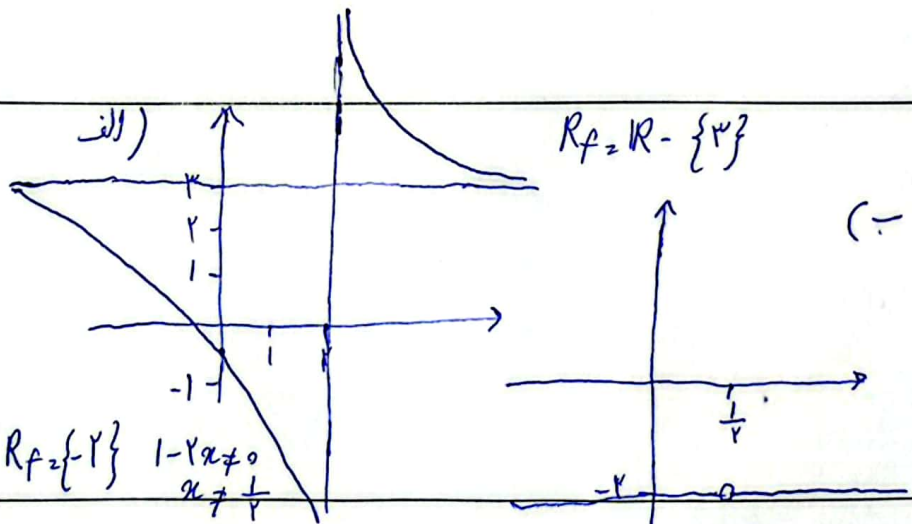
ب) $y = \sqrt{\frac{3x+1}{x+3}} \rightarrow R_f = \sqrt{\mathbb{R} - \{3\}} \rightarrow R_f = [0, \infty) - \{3\}$

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الف) $y = \frac{3x+1}{x-2}$

ب) $y = \frac{3x-2}{1-2x} =$

$\frac{-2(-2x+1)}{1-2x} = -2$ $R_f = \{3\}$ $1-2x \neq 0$
 $x \neq \frac{1}{2}$



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الف) $y = \cos^2 x + \frac{1}{\cos^2 x} \xrightarrow{a > 0} R_f = [2, \infty)$

ب) $y = 2\sqrt{\frac{x^2+1}{x}}$ $\frac{x^2+1}{x} = x + \frac{1}{x} \rightarrow R_f = (-\infty, -1] \cup [1, \infty)$

$R_f = (-\infty, -\sqrt{2}] \cup [\sqrt{2}, \infty)$

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