

الف) $f(x) = \sqrt{x+1} \rightarrow x+1 \geq 0 \quad D_f = [0, +\infty)$ $D_g = \mathbb{R} \quad D_f \neq D_g \rightarrow$ "تابع مرکب نیست"

ب) $f(x) = \frac{(x+3)(x+1)+x+1}{x^2+2x+1} \quad D_f = \mathbb{R} \quad g(x) = 1 \rightarrow D_g = \mathbb{R}$
 $x^2+2x+1 \rightarrow \Delta = 2^2 - (1 \times 1) = 0 \rightarrow$ "هوازیه" $\rightarrow \frac{x^2+2x+1}{x^2+2x+1} = 1 \quad f(x) = 1$ "تابع مرکب"

ج) $f(x) = \frac{2 \sin x (2 \sin x - 1)}{(2 \sin x - 1)}$ $D_f = \mathbb{R} \quad D_g(x) = \mathbb{R}$
 $2 \sin x \neq 1 \rightarrow \sin x \neq \frac{1}{2} = \sin \frac{\pi}{6} \rightarrow -1 \leq \sin x < 1$ "تابع مرکب نیست"

د) $D_f = \mathbb{R} - \{0\} \quad D_g = \mathbb{R} - \{0\}$ "تابع مرکب نیست"

ه) $f(x) = \left[\frac{x^2}{x^2+1} \right] = 0 \rightarrow 0 \leq x^2 < x^2+1 \rightarrow 0 \leq \frac{x^2}{x^2+1} < 1 \rightarrow D_f = \mathbb{R}$
 $g(x) = [x - [x]] \rightarrow 0 \leq x - [x] < 1 \rightarrow g(x) = 0 \quad D_g = \mathbb{R} \Rightarrow$ "تابع مرکب نیست"

و) $f(x) \rightarrow [2x] \neq 0 \quad 0 \leq 2x < 1 \rightarrow 0 \leq x < \frac{1}{2} \rightarrow D_f = \mathbb{R} - [0, \frac{1}{2})$
 $g(x) \rightarrow [x] \neq 0 \quad 0 \leq x < 1 \rightarrow D_g = \mathbb{R} - [0, 1)$ "تابع مرکب نیست"

ز) $f(x) \rightarrow x - |x| \geq 0 \rightarrow x \geq |x| \rightarrow D_f = \emptyset$ $g(x) \rightarrow |x| - x \geq 0 \rightarrow |x| \geq x \rightarrow D_g = \mathbb{R}$
 $D_g \neq D_f \rightarrow$ "تابع مرکب نیست"

ح) $D_g = \mathbb{R} \quad D_f = \mathbb{R} \rightarrow f(x) = g(x)$

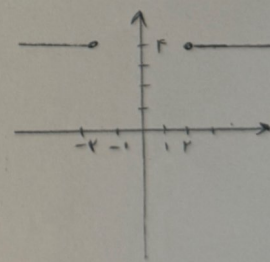
$f(x) = 2x^2 + 1 \rightarrow f(x) = x^2 + 1 \quad g(x) = x$

$f(x) - g(x) = (x^2 + 1)^2 - x^2 = x^4 + 2x^2 + 1 - x^2 = x^4 + x^2 + 1$

$f(x) \rightarrow x^2 - 1 \geq 0 \rightarrow x^2 \geq 1 \rightarrow x \geq 1 \quad D_f = [1, +\infty) \cup (-\infty, -1]$
 $x \leq -1$

$g(x) \rightarrow D_g = [1, +\infty) \cup (-\infty, -1]$ $f(x) = g(x)$

$g \circ f = (x - \sqrt{x^2 - 1})(x + \sqrt{x^2 - 1}) = x^2 - (x^2 - 1) = x^2 - x^2 + 1 = 1$



ا) $4m^2 - 2m - 2 = 0$
 $m^2 - m - 1 = 0$
 $(m-2)(m+1) = 0 \rightarrow m = \frac{2}{1} \quad m = -1$

$n = 1 \quad \frac{a+1}{-1} = \frac{1-b}{-1} \quad -4a-1 = -1+b$
 $b+1 \leq -4a$

$n = 0 \rightarrow 1 = -ab \rightarrow b = -1$

$a = \frac{1}{1}$

$am - bn = \frac{1}{1} \times \frac{1}{1} - (-1 \times \frac{1}{1})$

$m^2 - m + n = 0$

$n = \frac{1}{1} - \frac{1}{1} = 0$ $\frac{1}{1} - \frac{1}{1} + \frac{1}{1} = 1$

$1 + m + n = 0$

$m = -n - 1$

$1 - 1 + m + n = 0$

$1 + 1 + n + 1 + 1 + n = 0$

$4 + 2n = 0$

$n = -\frac{4}{2} = -2$

$m = \frac{1}{1} - \frac{1}{1} = 0$

$\frac{1}{1} - 1 = \frac{-1 + 1}{1} = 0$

$$\frac{ba+r}{\lambda a+b} = c \rightarrow ba+r = \lambda ca + cb$$

$$\lambda c = b$$

$$bc=r \rightarrow \lambda c^2 = r \rightarrow c^2 = \frac{r}{\lambda} \rightarrow c = \pm \frac{1}{\sqrt{\lambda}}$$

$$b = \pm r$$

$$\text{Case 1: } \lambda a + b = 0 \rightarrow \lambda a + b = 0$$

Case 1:

$$\lambda a = r$$

$$\lambda a = -r$$

$$a = \frac{1}{\sqrt{\lambda}}$$

$$a = -\frac{1}{\sqrt{\lambda}}$$

$$\frac{ab}{c} = \frac{-\frac{1}{\sqrt{\lambda}} - r}{-\frac{1}{\sqrt{\lambda}}} = -r$$

$$\frac{ab}{c} = \frac{\frac{1}{\sqrt{\lambda}} + r}{\frac{1}{\sqrt{\lambda}}} = r$$

$$\frac{ab}{c} = \pm r$$

$$g = \{(r, r), (r, r), (-1, -r), (r, r)\}$$

$$r^2 - 1 = \{(r, r), (r, r), (r, -r), (r, r)\}$$

$$rg = \{(r, r), (r, r), (-1, -r), (r, r)\}$$

$$f = \{(r, r), (r, r), (r, -r), (r, \frac{1}{r})\}$$

$$f+g = \{(r, r), (-1, -r), (r, r)\}$$

$$\frac{rg}{f+g} = \{(r, r), (-1, -r), (r, r)\}$$

$$\frac{R_{rg}}{f+g} = \{(r, r, r)\}$$

$$g = \{(r, -1), (-r, 1), (a, -rb), (a, 1)\}$$

$$f = \{(r, a), (-r, r^2 - b), (c, a), (r, d)\}$$

$$a = d = -1$$

$$ra - rb = 1 \rightarrow -r - rb = 1 \rightarrow -rb = r + 1 \rightarrow b = -r$$

$$a - rb = c \rightarrow -1 + r = c \rightarrow c = r - 1$$

$$d + c = -1 + r = r$$

$$-ar + m - m \geq 0 \rightarrow \Delta = 0 \rightarrow \Delta = b^2 - 4ac \leq 1 - r(-1)(-m) \leq 1 - rm$$

$$1 - rm \leq 0$$

$$m \geq \frac{1}{r}$$

$$-ar + m - \frac{1}{r} \geq 0 \rightarrow -(r - \frac{1}{r})^2 \geq 0$$

$$m = \frac{1}{r}$$

$$a = \frac{1}{r}, b = 0$$

$$a + b = 0 + \frac{1}{r} = \frac{1}{r}$$

$$0 < \lambda < \infty \rightarrow \lambda = c$$

$$\frac{r}{m-c} = \frac{r(m+a)}{m^2 + 4m + b}$$

$$rm^2 + 4rm + rb = rm^2 + am - rac - ac$$

$$1' = -rac + a \rightarrow a - rc = 1'(\star)$$

$$-ac = 1'b(\star\star)$$

$$x = 0 \rightarrow \frac{r}{-c} = \frac{a}{b}$$

$$(\star) \rightarrow -rc = 1'$$

$$c = -r, b = 1$$

$$a = r$$

$$\frac{r}{m-c} = \frac{r(m+\frac{a}{r})}{(m+\frac{a}{r})(m-c)} = \frac{r}{m-c}$$

$$c = -\frac{a}{r} \rightarrow -a = rc$$

$$a = -rc(\star\star)$$

$$P_p = R - \left[-\frac{a}{r}, c\right]$$

$$a + b + c = r + 1 - r = 1$$