

۱	الف) $f(x) = \sqrt{x+1}$, $g(x) = x \rightarrow D_f = \mathbb{R}^+ + \{0\}$ و $D_g = \mathbb{R}$ ✓ بارهستند ب) $f(x) = \frac{(x+1)(x+1) + x+1}{x^2 + x + 1}$, $g(x) \textcircled{1} \rightarrow f(x) = \frac{x^2 + 2x + 2}{x^2 + x + 1} \textcircled{1}$ ✓ بارهستند (مطابق باشد) ج) $f(x) = \frac{x \sin^2 x - 4 \sin x}{x \sin^2 x + 4 \sin x}$, $g(x) = x \sin x \rightarrow D_f = \mathbb{R}$, $D_g = \mathbb{R}$ ✓ بارهستند (مطابق باشد) د) $f(x) = \frac{x}{ x }$, $g(x) = \frac{ x }{x} \rightarrow D_f = \mathbb{R} - \{0\}$, $D_g = \mathbb{R} - \{0\}$ ✓ بارهستند (مطابق باشد)
۲	الف) $f(x) = \left[\frac{x^2}{x^2+1} \right]$, $g(x) = [x - [x]] \rightarrow D_f = \mathbb{R}$, $D_g = \mathbb{R}$ ✓ بارهستند (مطابق باشد) ب) $f(x) = \frac{1}{[x]}$, $g(x) = \frac{1}{x}$ ✓ بارهستند $\Rightarrow x = \frac{1}{x} \rightarrow f(x) = 1$, $g(x) = \frac{1}{x} \rightarrow$ تقریب شد ✓ بارهستند ج) $f(x) = \frac{1}{\sqrt{x-1}}$, $g(x) = \frac{1}{\sqrt{x+1}}$ ✓ بارهستند د) $f(x) = \frac{x^2 - x}{x-1}$, $g(x) = \frac{x(x^2-1)}{x-1} = \frac{x(x-1)(x+1)}{x-1} = x^2 + x$, $g(x) = x^2 + x \rightarrow D_f = \mathbb{R}$, $D_g = \mathbb{R}$ ✓ بارهستند (مطابق باشد)
۳	$f(x) + g(x) = x^2 + x + 1$ $f^2(x) - g^2(x) = (x^2+1)^2 - x^2$ $f(x) - g(x) = x^2 - x + 1$ $= x^2 + 1 + x^2 - x^2 = x^2 + 1$ $\begin{cases} f(x) + g(x) = x^2 + x + 1 \\ f(x) - g(x) = x^2 - x + 1 \end{cases} \rightarrow \begin{cases} 2f(x) = 2x^2 + 2 \\ 2g(x) = x \end{cases}$ $f(x) = x^2 + 1$, $g(x) = x$ ✓
۴	$D_f = D_g \rightarrow x^2 \geq 1 \rightarrow x \geq 1$, $x \leq -1 \rightarrow D_f = D_g = D_{f \times g} = (-\infty, -1] \cup [1, +\infty)$ $f \times g = (x - \sqrt{x^2 - 1})(x + \sqrt{x^2 - 1})$ ✓
۵	$ax + r = k(x-b)$, $x^2 - mx + n = k(rx^2 - px - a)$ $am = \frac{1}{r} \times \frac{r}{r} = \frac{r}{r}$ از ضرایب x^2 در معادله $bn = -rx - \frac{a}{r} = 1$ از ضرایب x در معادله $am - bn = \frac{r}{r} - 1 = 0 = \frac{-r}{r}$ از صورت x در معادله $\begin{cases} n = \frac{1}{r}(-a) = \frac{-a}{r} , m = \frac{1}{r}(-r) = \frac{-r}{r} \\ r = -\frac{1}{r}b \rightarrow b = -r , a = \frac{1}{r} \end{cases}$ ✓

$Dg: \mathbb{R} \setminus \{a\} \rightarrow a = -\frac{b}{\lambda}$ $\rightarrow b + \lambda x = c(\lambda x + b) \rightarrow b + \lambda c = c^2 + cb \rightarrow c = c(\lambda c) + \lambda c^2 \rightarrow c^2 + \frac{1}{c} = c + \frac{1}{c}$ if $c = \frac{1}{t} \rightarrow b + \lambda c = t, a = -\frac{b}{\lambda} = -\frac{t}{\lambda} = -\frac{1}{t} \rightarrow \frac{ab}{c} = \frac{-\frac{1}{t} \times t}{\frac{1}{t}} = -t$ if $c = -\frac{1}{t} \rightarrow b + \lambda c = -t, a = -\frac{b}{\lambda} = \frac{t}{\lambda} = \frac{1}{t} \rightarrow \frac{ab}{c} = \frac{\frac{1}{t} \times t}{-\frac{1}{t}} = -t$	6
$f(x) = \{(1, 2), (2, 4), (3, 6), (0, \frac{1}{2})\} \rightarrow rg = \{(1, 2), (2, 4), (-1, -2), (0, 0)\}$ $g(x) = \{(2, 4), (3, 6), (-1, -2), (0, 0)\} \rightarrow f+g = \{(2, 4), (3, 6), (-1, -2)\}$ $\frac{rg}{f+g} = \{(2, \frac{2}{1}), (3, \frac{3}{1}), (-1, \frac{-1}{-1})\} \Rightarrow \{(2, 1), (3, 1), (-1, 1)\}$	7
$a = d = -1$ $3a - 2b = 1 \rightarrow 3(-1) - 2b = 1 \rightarrow b = -2$ $a - 2b = c \rightarrow -1 - 2(-2) = c \rightarrow c = 3$ $d + c = -1 + 3 = 2$	8
$f(x) = \sqrt{-x^2 + 4x + 5} \rightarrow \Delta = 0$ $-x^2 + 4x + 5 = 0 \rightarrow \Delta = 0 \rightarrow b^2 - 4ac = 0 \rightarrow 16 - 4 \times 1 \times 5 = 0 \rightarrow m = \frac{1}{t}$ $-x^2 + 4x - \frac{1}{t} = 0 \rightarrow x = \frac{-b \pm \sqrt{\Delta}}{2a} = \frac{-4 \pm \sqrt{0}}{-2} = \frac{-4}{-2} = 2 \rightarrow a = \frac{1}{t}$ $a + b = \frac{1}{t} + 0 = \frac{1}{t}$	9
$Dg: \mathbb{R} \setminus \{c\} \rightarrow Dg: \mathbb{R} \setminus \{c\} \rightarrow \Delta = 0$ $x^2 + 4x + b = 0 \rightarrow b^2 - 4ac = 0 \rightarrow 16 - 4b = 0 \rightarrow b = 4$ $(x + 2)^2 = 0 \rightarrow x = -2 \rightarrow c = -2$ $\frac{x^2 + 4}{x^2 + 4x + 4} = \frac{x}{x + 2} \rightarrow x^2 + 4 = x(x + 2) \rightarrow x^2 + 4 = x^2 + 2x \rightarrow 4 = 2x \rightarrow x = 2$ $\Rightarrow a + b + c = 4 + 4 - 2 = 6$	10