

الف) $Df: \mathbb{R} \setminus \{0\} \rightarrow \mathbb{R} \setminus \{0\} \rightarrow Df = [0, +\infty), Dg = \mathbb{R} \quad \times$

سوال (۱)

ب) $f(x) = \frac{x^2 + 2x + 1}{x^2 + 2x + 1} = 1, g(x) = 1 \Rightarrow Df, Dg = \mathbb{R} \quad \checkmark$

ج) $f(x) = \frac{2 \sin x (2 \sin x - 1)}{2 \sin x - 1} = 2 \sin x, g(x) = 2 \sin x \Rightarrow Df = Dg = \mathbb{R} \quad \checkmark$

د) $f(x) = \frac{x}{|x|} \rightarrow Df = \mathbb{R} \setminus \{0\}, g(x) = \frac{|x|}{x} \rightarrow Dg = \mathbb{R} \setminus \{0\}, Df = Dg \quad \checkmark$

سوال (۲)

الف) $f(x) = \left[\frac{x^2}{x^2 + 1} \right], g(x) = [x - [x]] \Rightarrow Df = Dg = \mathbb{R} \quad \checkmark$

ب) $\frac{1}{[x]} \rightarrow [x] \neq 0 \rightarrow x \in [0, 1) \rightarrow Df = (-\infty, 0) \cup [\frac{1}{x}, +\infty) \quad \times$

$x[x] \neq 0 \rightarrow [x] \neq 0 \rightarrow x \in [0, 1) \rightarrow Dg = (-\infty, 0) \cup [1, +\infty)$

ج) $f(x) = \frac{1}{\sqrt{x-|x|}}, g(x) = \frac{x}{x-|x|} \Rightarrow Df = \emptyset, Dg = (-\infty, 0) \quad \times$

د) $f(x) = \frac{x^2 - x}{x - 1} = \frac{x(x-1)}{x-1} = x(x+1) = x^2 + x \Rightarrow Df = \mathbb{R} = Dg \quad \checkmark$

$f(x) + g(x) = x^2 + x + 1, f(x) - g(x) = x^2 - x + 1$

سوال (۳)

$f'(x) - g'(x) = ? \quad f(x) = ? \quad g(x) = ?$

$\begin{cases} f(x) + g(x) = x^2 + x + 1 \\ f(x) - g(x) = x^2 - x + 1 \end{cases} \rightarrow 2f(x) = 2x^2 + 2 \Rightarrow f(x) = x^2 + 1$

$g(x) = x \quad f'(x) = (x^2 + 1)' = 2x, g'(x) = x$

$f'(x) - g'(x) = 2x - x = x$

سوال (۴)

$f(x) = x - \sqrt{x^2 - 1}, g(x) = x + \sqrt{x^2 - 1}$

؟ $f \times g$

$Df \rightarrow x^2 - 1 \geq 0 \rightarrow Df = (-\infty, -1] \cup [1, +\infty)$

$Dg \rightarrow x^2 - 1 \geq 0 \rightarrow Dg = (-\infty, -1] \cup [1, +\infty)$

$f \times g = (-\infty, -1] \cup [1, +\infty) \quad x^2 - (x^2 - 1) = 1$

$$x^2 - mx + n = r(x^2 - rx - 0) = x^2 - mx + n = rx^2 - rx - \omega r \quad (\text{سوال 5})$$

$$ax + r = r(x - b) = \frac{1}{r}(x - b) \rightarrow a = \frac{1}{r} \quad (\text{ضرب في } r), \quad -\frac{1}{r}b = r \Rightarrow b = -r \quad (\text{سوال 6})$$

$$am - bn = \left(\frac{1}{r} \times \frac{r}{r}\right) - \left(-\frac{r}{r} \times -\frac{\omega}{r}\right) = \frac{r}{r} - 1 = -\frac{r\omega}{r}$$

$$1 = r\omega \Rightarrow r = \frac{1}{\omega} \quad -m = -r\omega \Rightarrow +\frac{r\omega}{r} = m \quad n = -\omega r \Rightarrow n = -\frac{\omega}{\omega} \quad (\text{سوال 7})$$

$$\Delta x + b \neq 0 \rightarrow x \neq -\frac{b}{\Delta} \Rightarrow a = -\frac{b}{\Delta} \quad b\Delta x + r = (\Delta x + b)(c) \Rightarrow b\Delta x + r = \Delta xc + bc$$

$$b = \Delta c \quad c = \frac{b}{\Delta} \quad / \quad r = bc \rightarrow c = \frac{r}{b} \quad / \quad r = bc = \Delta c^2 \Rightarrow c^2 = \frac{1}{\Delta} \Rightarrow c = \pm \frac{1}{\sqrt{\Delta}}$$

$$a = -\frac{b}{\Delta} = -\frac{\Delta c}{\Delta} = -c \rightarrow \frac{ab}{c} = \frac{-cb}{c} = -b \rightarrow \Delta c = \frac{1}{\Delta} \rightarrow -\Delta = \frac{1}{\Delta} = -\Delta$$

$$\Delta c = -\frac{1}{\Delta} \rightarrow +\Delta x + \frac{1}{\Delta} = r$$

$$f = \left\{(-1, r)(r, f)(r, -r)(0, +\frac{1}{r})\right\}$$

$$g = \left\{(r, f)(r, g)(-1, -f)(\omega, 0)\right\} \rightarrow \text{مثال} = \frac{r\omega}{f \cdot g} \quad B = \{f, r, 1\} \quad (\text{سوال 12})$$

$$g = \{(r, -1)(-r, 1)(a - rb, \omega)\}, \quad f = \{(r, a)(-r, ra - rb)(c, \omega)(r, d)\} \quad (\text{سوال 13})$$

$$a = d = -1 \quad / \quad \frac{ra}{c - r} - rb = 1 \rightarrow -rb = f \Rightarrow b = -r \quad \frac{a - rb}{c} = c \Rightarrow c = \omega$$

$$d + c = -1 + \omega = \boxed{f}$$

(سوال 14)

$$-x^2 + x - m > 0 \rightarrow \Delta = b^2 - 4ac \rightarrow 1 - 4m > 0 \rightarrow 1 - 4m = 0 \rightarrow 4m = 1 \Rightarrow m = \frac{1}{4}$$

$$-x^2 + x - \frac{1}{4} \rightarrow x = \frac{-b}{2a} = \frac{-1}{-2} = \frac{1}{2}, \quad f\left(\frac{1}{2}\right) = 0$$

$$g(x) = \left\{\left(\frac{1}{2}, 0\right)\right\} \rightarrow a + b = \boxed{\frac{1}{2}}$$

(سوال 15)

$$\text{مجموعه } = (rx + a)(x - c) = r(x^2 + 4x + b) \rightarrow rx^2 - rx + ax - ac = rx^2 + 4rx + rb$$

$$-rx + ax - ac = 4rx + rb \rightarrow x(a - rc) - ac = 4rx + rb \rightarrow a - rc = 4r / -ac = rb \Rightarrow b = -\frac{ac}{r}$$

$$(x - c)^2 \cdot x^2 + 4x + b \rightarrow x^2 - 2xc + x^2 = x^2 + 4x + b \Rightarrow -2c = 4 \Rightarrow c = -2, \quad b = 9$$

$$a - rc = 4r \rightarrow a = 4 \quad a + b + c = \boxed{12}$$