

Subject:

Year:

Month:

Day:

الف) $f(x) = \sqrt{x|x|}$ و $g(x) = x \rightarrow D_f \neq D_g \Rightarrow$ مساوی نیست

$\begin{matrix} + \sqrt{x} \\ - \sqrt{x} \\ 0 \sqrt{x} \end{matrix} \rightarrow D_f = [0, +\infty) \quad D_g = \mathbb{R}$

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ب) $f(x) = \frac{(x+1)(x+1)+x+1}{x^2+2x+1}$ و $g(x) = 1 \rightarrow D_f = D_g \Rightarrow$ مساوی است

$x^2+2x+1 \neq 0 \rightarrow D_g = \mathbb{R}$

$\Delta = 2^2 - 4 = 0 \rightarrow D_f = \mathbb{R}$

ج) $f(x) = \frac{r \sin^2 x - r \sin x}{r \sin x - r}$ و $g(x) = r \sin x \Rightarrow D_f = D_g \Rightarrow$ مساوی است

$r \sin x - r \neq 0 \rightarrow D_g = \mathbb{R}$

$\sin x \neq \frac{r}{r} \rightarrow -1 \leq \sin x \leq 1$

$x = 0$

د) $f(x) = \frac{x}{|x|}$ و $g(x) = \frac{|x|}{x} \Rightarrow D_f = D_g \Rightarrow$ مساوی است

$|x| \neq 0 \Rightarrow D_f = \mathbb{R} - \{0\}$

$x \neq 0 \Rightarrow D_g = \mathbb{R} - \{0\}$

ه) $f(x) = \left[\frac{x^r}{x^r+1} \right]$ و $g(x) = [x - [x]] \rightarrow D_f = D_g \Rightarrow$ مساوی است

$D_g = \mathbb{R} \rightarrow 0 \leq x - [x] < 1 \rightarrow [x - [x]] = 0$

$0 \leq \frac{x^r}{x^r+1} < 1 \rightarrow 0 = \left[\frac{x^r}{x^r+1} \right]$

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و) $f(x) = \frac{1}{[rx]}$ و $g(x) = \frac{1}{r[x]}$ $\rightarrow D_f \neq D_g$

$[rx] \neq 0 \rightarrow r[x] \neq 0 \rightarrow [x] \neq 0 \rightarrow 0 \leq x < 1$

$0 \leq rx < 1 \rightarrow 0 \leq x < \frac{1}{r} \rightarrow D_f = \mathbb{R} - [0, \frac{1}{r}) \quad D_g = \mathbb{R} - [0, 1)$

ز) $f(x) = \frac{1}{\sqrt{x-|x|}}$ و $g(x) = \frac{1}{\sqrt{|x|-x}}$ $\rightarrow D_f \neq D_g \rightarrow$ مساوی نیست

$x - |x| > 0 \rightarrow \begin{matrix} +x \\ -x \\ 0 \end{matrix} \rightarrow D_f = \emptyset$

$|x| - x > 0 \rightarrow \begin{matrix} +x \\ -x \\ 0 \end{matrix} \rightarrow D_g = (-\infty, 0)$

$$2) f(x) = \begin{cases} \frac{x^r - x}{x-1}, & x \neq 1 \\ r, & x = 1 \end{cases}$$

$$g(x) = x^r + x = x(x+1) \rightarrow Dg = \mathbb{R}$$

$$Dg = Df \Rightarrow \text{بله}$$

$$\frac{x^r - x}{x-1} = \frac{x(x^{r-1} - 1)}{x-1} = \frac{x(x+1)(x^{r-2} - x^{r-3} + \dots - 1)}{x-1} = x(x+1)$$

نقطه شکی تو این هست که فرقی بین
در صورت $x=1$ میزنو تعریف نشده یعنی
ولی چون خودش نقطه $x=1$ پس مشکلی نیست $\rightarrow D_f = \mathbb{R}$

$$\begin{cases} f(x) + g(x) = x^r + x + 1 \\ f(x) - g(x) = x^r - x + 1 \end{cases}$$

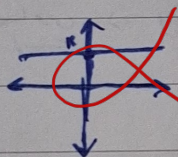
$$\rightarrow 2f(x) = 2x^r + 2 \rightarrow \boxed{f(x) = x^r + 1}$$

$$\rightarrow x^r + 1 + g(x) = x^r + x + 1 \rightarrow \boxed{g(x) = x}$$

$$\rightarrow f'(x) - g'(x) = (x^r + 1)' - x = x^r + 1 - 1 = x^r = f'(x) - g'(x)$$

$$f(x) = x - \sqrt{x^2 - r} \rightarrow f \circ g = (x - \sqrt{x^2 - r})(x + \sqrt{x^2 - r}) = x^2 - x^2 + r = r$$

$$g(x) = x + \sqrt{x^2 - r}$$



دامنه $f \circ g$ ؟

شکل با دامنه عادی $\rightarrow$

$$f(x) = \frac{ax+r}{x^r - mx + n} = g(x) = \frac{x-b}{x^r - rx - d}$$

$$\rightarrow x^r - mx + n = (x - \frac{d}{r})(x+1) \rightarrow rx^r - rx - d \neq 0 \Rightarrow x^r - rx - 1 \neq 0 \Rightarrow (x-d)(x+r) \neq 0$$

$$x^r - \frac{r}{r}x - \frac{d}{r} = \frac{r}{r}x - \frac{d}{r} \Rightarrow n = -\frac{d}{r} \quad mx = \frac{r}{r}x = m = \frac{r}{r} \rightarrow x \neq -\frac{d}{r}, x \neq -\frac{r}{r} = -1$$

$$\frac{ax+r}{x^r - \frac{r}{r}x - \frac{d}{r}} = \frac{x-b}{x^r - rx - d} \Rightarrow \frac{ax+r}{x^r - \frac{r}{r}x - \frac{d}{r}} = \frac{\frac{x}{r} - \frac{b}{r}}{x^r - \frac{r}{r}x - \frac{d}{r}} \rightarrow ax+r = \frac{x}{r} - \frac{b}{r}$$

$$\rightarrow ax = \frac{1}{r}x - \frac{b}{r} \rightarrow \boxed{a = \frac{1}{r}, b = -r}$$

$$\rightarrow r = \frac{-b}{r} \rightarrow \boxed{b = -r}$$

$$\rightarrow am - bn = (\frac{1}{r} \times \frac{r}{r}) - ((-1) \times (\frac{-d}{r})) = \frac{r}{r} - 1 = \frac{-rv}{r}$$

$$f(x) = \frac{bx+r}{ax+b} \quad g(x) = c \Rightarrow Dg = \mathbb{R} - \{a\}$$

$$\rightarrow Df = Dg \rightarrow \Lambda a + b = 0 \rightarrow f(x) = c \rightarrow \frac{bx+r}{\Lambda x + b} = c \rightarrow bx+r = \Lambda cx + bc \rightarrow \Lambda c = b$$

$$a = \pm \frac{1}{r} \leftarrow \Lambda a = \pm r$$

$$\boxed{c = -\frac{1}{r} \rightarrow b = -r \rightarrow a = \frac{1}{r}}$$

دامنه جواب با r درست

$$f(x) = \frac{-rx+r}{\Lambda x - r} = -\frac{1}{r} = c \quad b = \pm r$$

$$Df = \mathbb{R} - \{\frac{1}{r}\} = \mathbb{R} - \{a\}$$

$$\boxed{c = \frac{1}{r} \rightarrow b = r \rightarrow a = -\frac{1}{r}} \rightarrow f(x) = \frac{rx+r}{\Lambda x + r} = \frac{1}{r} = c$$

$$f(x) = \mathbb{R} - \{\frac{1}{r}\} = \mathbb{R} - \{a\}$$

$$yf-1 = \{(-1, r)(r, v)(r, -d)(0, 0)\}$$

✓ -v

$$yf = \{(-1, r)(r, \wedge)(r, -r)(0, 1)\} \Rightarrow f = \{(-1, r)(r, r)(r, -r)(0, \frac{1}{r})\}$$

$$g = \{(r, r)(r, r)(-1, -r)(d, 0)\} \Rightarrow rg = \{(r, \wedge)(r, r)(-1, \wedge)(d, 0)\}$$

$$\frac{rg}{f+g} = \{(-1, r)(r, 1)(r, r)\}$$

$$f = \{(r, a)(-r, ra-rb)(c, d)(r, d)\} \rightarrow \boxed{a=d=-1}$$

✓ -\wedge

$$g = \{(r, -1)(-r, 1)(a-rb, d)\} \rightarrow ra-rb=1 \Rightarrow -r-rb=1 \rightarrow \boxed{b=-r}$$

$$a-rb=c \rightarrow -1-r(-r) = -1+r = \boxed{d=c} \quad d+c = -1+d = \boxed{r}$$

✓ -r

$$f(x) = \sqrt{-x^r + x - m} = g(x) = \{(a, b)\}$$

$$-x^r + x - m > 0 \quad \Delta = 1 - 4m \Rightarrow \Delta = 0 \Rightarrow 1 - 4m = 0 \Rightarrow m = \frac{1}{4}$$

$$f(\frac{1}{r}) = \sqrt{\frac{1}{r} + \frac{1}{r} - \frac{1}{r}} = \boxed{0=b} \quad a+b = \frac{1}{r} + 0 = \frac{1}{r} \quad \rightarrow f(g) = \sqrt{-(x-\frac{1}{r})^r} \rightarrow x = \frac{1}{r} \rightarrow Dg = \{\frac{1}{r}\} \rightarrow \boxed{a=\frac{1}{r}}$$

$$f(x) = \frac{rx+a}{x^r+4x+b} = g(x) = \frac{r}{x-c}$$

$$\Delta = r^2 - 4b = 0 \rightarrow \boxed{b=r^2}$$

$$x^r+4x+r = (x+r)^r \Rightarrow \boxed{x=-r}$$

$$Df = \mathbb{R} - \{-r\} \rightarrow \boxed{c=-r}$$

$$Dg = \mathbb{R} - \{c\}$$

$$\frac{rx+a}{(x+r)^r} = \frac{r}{x+r}$$

$$rx+4 = rx+a \rightarrow \boxed{a=4}$$

$$a+b+c = 4+r-r = \boxed{1r}$$

✓