سوال ۱) مقدار کمتر تابع را دارد

الف) $f(x) = x|x| \geq 0 \to x \geq 0 \to x^2 \geq 0$ $f(x) = \sqrt{x|x|} = \sqrt{x \cdot x} = \sqrt{x^2} = |x| = x$

$f(x) = x = g(x)$ $f(x) = [0, +\infty)$ $g(x) = R$ ✗ برابر نیست فقط در $[0, +\infty)$ برابرند

ب) $f(x) = \dfrac{(x+3)(x+1) + x + 4}{x^2 + 5x + 7} \to f(x) = \dfrac{x^2 + 4x + 3 + x + 4}{x^2 + 5x + 7} = \dfrac{x^2 + 5x + 7}{x^2 + 5x + 7}$

$g(x) = 1$

دلتا $5^2 - 4 \times 7$

✓ برابر است $Df = R$ $Dg = R$ $Df = Dg$ یعنی مخرج صفر نمی‌شود ← دلتا منفی $25 - 28 = -3$

ج) $f(x) = \dfrac{2\sin x(2\sin x - 3)}{2\sin - 3} = 2\sin x$ $f(x) = 2\sin x = g(x)$

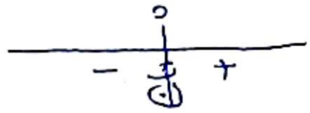
$2\sin x - 3 = 0 \to 2\sin x = 3 \to \sin = \frac{3}{2}$

$\frac{3}{2} > 1$ $Df = R$ $Df = Dg$

$Dg = R$ برابر است ✓

د) $f(x) = \dfrac{x}{|x|}$, $g(x) = \dfrac{|x|}{x}$

$x > 0 \to f(x) = \frac{x}{x} = 1$, $g(x) = \frac{x}{x} = 1$



$x < 0 \to f(x) = \frac{x}{-x} = -1$, $g(x) = \frac{-x}{x} = -1$ $Df = Dg$ برابر است ✓

سوال ۲)

الف) $f(x) = \left[\dfrac{x^2+1-1}{x^2+1}\right] = \left[1 - \dfrac{1}{x^2+1}\right] = \left[1^-\right] = 0$ $g(x) = [x - [x]] = 0$ ثابت صفر ✓

$Df = Dg$ برابر

ب) $f = \dfrac{1}{[2x]}$ $Df = [2x] \neq 0$ $x \neq [0, \frac{1}{2})$

$Dg = 2[x] \neq 0$ $[x] \neq 0$ $x \neq [0, 1)$ $Df \neq Dg$ برابر نیست ✗

ج) $Df \neq \emptyset$ $Dg = (-\infty, 0)$ $Df \neq Dg$ برابر نیست ✗

د) $f(x) = \begin{cases} x^2 + x & x = 1 \\ 2 & x \neq 1 \end{cases}$ $g(x) = x^2 + x$ $g(1) = f(1)$ برابر است ✓

$Df = Dg = R$ $g(x) = f(x)$

سوال ۳)

$$f^2(x)-g^2(x)=\big(f(x)-g(x)\big)\big(f(x)+g(x)\big)\rightarrow$$

$$(x^2-x+1)(x^2+x+1)=x^4+x^2+1$$

$$f(x)-g(x)=x^2-x+1$$

$$f(x)+g(x)=x^2+x+1$$

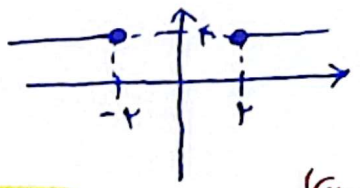
$$\longrightarrow \begin{cases} f(x)=x^2+1 \\ g(x)=x \end{cases}$$

سوال ۴)

$$f(x)=x-\sqrt{x^2-4} \longrightarrow x^2-4\geq 0 \rightarrow x^2\geq 4 \begin{cases} x\geq 2 \\ x\leq -2 \end{cases} \quad Df=(-\infty,-2]\cup[2,+\infty)$$

$$g(x)=x+\sqrt{x^2-4} \longrightarrow x^2-4\geq 0 \rightarrow x^2\geq 4 \begin{cases} x\geq 2 \\ x\leq -2 \end{cases} \quad Dg=(-\infty,-2]\cup[2,+\infty)$$

$$Df\times g=(-\infty,-2]\cup[2,+\infty) \qquad f\times g=x^2-(x^2-4)=4$$



سوال ۵)

$$\frac{ax+2}{x^2-mx+n}=\frac{x-b}{2x^2-3x-5}=2ax^2-3ax^2-5ax+4x^2-4x-10=$$

$$2\left(x^2-\frac{3}{2}x-\frac{5}{2}\right)\longrightarrow \begin{matrix} m=\frac{3}{2} \\ n=-\frac{5}{2} \end{matrix} \longrightarrow \frac{ax+2}{x^2-\frac{3}{2}x-\frac{5}{2}}=\frac{x-b}{2\left(x^2-\frac{3}{2}x-\frac{5}{2}\right)}$$

$$2ax+4=x-b \longrightarrow a=\frac{1}{2} \quad b=-2 \qquad am-bn=\frac{3}{4}-10=-\frac{37}{4}$$

سوال ۶)

$ac = b$

$bc = 1 \rightarrow ac^2 = 4 \rightarrow c = \pm\frac{1}{2}$

$\rightarrow c = \frac{1}{2} \rightarrow a = -\frac{1}{2} \rightarrow b = 4 \rightarrow \frac{ab}{c} = -4$

$\rightarrow c = -\frac{1}{2} \rightarrow a = \frac{1}{2} \rightarrow b = -4 \rightarrow \frac{ab}{c} = +4$

سوال ۷

$\frac{2g}{f+g} = \frac{\{(2,8),(3,12),(-1,-8),(5,20)\}}{\{(2,8),(3,4),(-1,-2)\}} = \{(2,1),(3,4),(-1,4)\}$

$R = \{(2,1),(3,4),(-1,4)\}$

سوال ۸)

$a = -1$

$-2 - 2b = 11 \rightarrow -2b = 4 \rightarrow b = -2$ $\qquad a + c = 4$

$a - 2b = c \rightarrow -1 - 2(-2) = 5 \rightarrow c = 5$

سوال ۹)

$f(x)=\sqrt{-x^2+x-m} \Rightarrow -x^2+x-m \geq 0 \rightarrow -x^2+x \geq m \rightarrow -x(x-1) \geq m$

$\downarrow$

$\Delta < 0 \rightarrow m > \frac{1}{4}$ $\quad f(x)=\sqrt{-(x-\frac{1}{2})^2} \rightarrow a=\frac{-1}{2}$ $b=0$

$a+b=\frac{1}{2}$

سوال ۱۰)

$f(x)=g(x) \rightarrow \frac{2x+a}{x^2+4x+b}=\frac{2}{x-c}$

$\downarrow$

$\Delta=0 \rightarrow b=9 \rightarrow D_f=D_g \rightarrow c=-2$

$\frac{2x+a}{x+2}=2 \rightarrow a=4$ $\quad a+b+c=12$