(۱)

الف) $f(x)=\sqrt{x|x|}$, $g(x)=x$

if $x=-2$ $f(-2)=$ تعریف نشده ، $g(-2)=-2$ برابر نیست ، ✗

ب) $f(x)=\frac{\overbrace{(x+3)(x+1)}^{x^2+4x+3}+x+4}{x^2+5x+7}$, $g(x)=1$ $f(x)=\frac{x^2+5x+7}{x^2+5x+7}=1 \Rightarrow$

$f(x)=1$ مساوی هستند ✓

ج) $f(x)=\frac{4\sin^2 x-6\sin x}{2\sin x-3}$, $g(x)=2\sin x$

$f(x)=\frac{2\sin x\cancel{(2\sin x-3)}}{\cancel{(2\sin x-3)}}=2\sin x$ برابرند ✓

د) $f(x)=\frac{x}{|x|}$, $g(x)=\frac{|x|}{x}$

$f(-2)\to\frac{-2}{2}=-1$ $g(-2)=\frac{2}{-2}=\frac{-2}{2}=-1$ برابرند ✓

$D_f=\mathbb{R}-\{0\}$ $D_g=\mathbb{R}-\{0\}$

(۲)

الف) $f(x)=\left[\frac{x^2}{x^2+1}\right]$, $g(x)=[x-[x]]$

$D_f=\mathbb{R}$ $D_g=\mathbb{R}$

$x\in\mathbb{Z}\to x-[x]=0$

$x\notin\mathbb{Z}\to x-[x]=0$

$g(x)\underset{\text{همواره}}{=}0$

$x^2<x^2+1$

$\left[\frac{x^2}{x^2+1}\right]=0$

$f(x)\underset{\text{همواره}}{=}0$

برابرند ✓

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ب) $f(x)=\frac{1}{[2x]}$, $g(x)=\frac{1}{2[x]}$

$x=\frac{3}{4}$

$\to f(x)=\frac{1}{[2\times\frac{3}{4}]}=\frac{1}{[\frac{3}{2}]}=\frac{1}{1}=1$

برابر نیستند ✗

$\to g(x)=\frac{1}{2[\frac{3}{4}]}=\frac{1}{2\times 0}=\frac{1}{0}$ تعریف نشده

ج) $f(x)=\frac{1}{\sqrt{x-|x|}}$, $g(x)=\frac{1}{\sqrt{|x|-x}}$

$D_f=\mathbb{R}^+$ $D_g=\mathbb{R}^-$ برابر نیستند ✗

$f(-2)=\frac{1}{\sqrt{(-2)-|2|}}=\frac{1}{\sqrt{-4}}=$ تعریف نشده $g(-2)=\frac{1}{\sqrt{|-2|+2}}=\frac{1}{2}$

د) $f(x)=\begin{cases}\frac{x^3-x}{x-1} & ; x\neq 1\\ 2^{x-1} & ; x=1\end{cases}$, $g(x)=x^2+x$

$\frac{x^3-x}{x-1}=\frac{x(x^2-1)}{(x-1)}=\frac{x\cancel{(x-1)}(x+1)}{\cancel{(x-1)}}=x(x+1)$

برابرند ✓

$g(x)=x(x+1)$

$f(1)=2$
$g(1)=1+1=2$

۳)

$\begin{cases}f(x)+\cancel{g(x)}=x^2+\cancel{x}+1\\ f(x)-\cancel{g(x)}=x^2-\cancel{x}+1\end{cases}\Longrightarrow 2f(x)=2x^2+2\to\boxed{f(x)=x^2+1}$

$\cancel{x^2}+\cancel{1}+g(x)=\cancel{x^2}+x+\cancel{1}\to\boxed{g(x)=x}$

$f^2(x)-g^2(x)\to(x^2+1)^2-(x)^2=x^4+1+2x^2-x^2=\boxed{x^4+x^2+1}$

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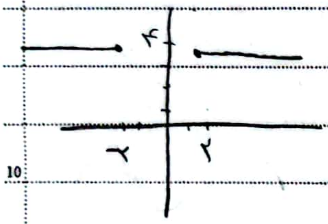
$f(x) = x - \sqrt{x^2-4} \to x - \overbrace{\sqrt{x^2-4}}^{a}$

$g(x) = x + \sqrt{x^2-4} \to x + \underbrace{\sqrt{x^2-4}}_{a}$

$x^2 - \left(\sqrt{x^2-4}\right)^2 \to x^2 - (x^2-4) = x^2 - x^2 + 4 = \boxed{4}$



x باید متعلق به دامنه باشد

$f \times g = (x-a)(x+a) = x^2 - a^2$

$D_f = \sqrt{x^2-4} = \sqrt{(x-2)(x+2)}$ → ۲ ، -۲

-۲		۲	
+	○ −	○	+

$$D_g = D_f = (-\infty, -2] \cup [2, +\infty)$$
$$f \times g = 4$$

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$f(x) = g(x)$

$g(x) = \frac{x-b}{2x^2-3x-5} \to 2x^2-3x-5=0 \to x^2-3x-10=0 \to (x+2)(x-5)=0$

ریشه‌ها: $\frac{-2}{2} = -1$ ، $\frac{5}{2}$

ریشه‌های g(x) را پیدا می‌کنیم و آن عددها باید ریشه‌ی f(x) هم باشند.

$f(x) = \frac{ax+2}{x^2-mx+n} \to x^2 - mx + n \xrightarrow{\boxed{x=-1}} (1+m+n=0) \times (-1)$

$\to x^2 - mx + n \xrightarrow{\boxed{x=\frac{5}{2}}} \frac{25}{4} - \frac{5}{2}m + n = 0$

$$\begin{cases} -m - \cancel{n} = 1 \\ -\frac{5}{2}m + \cancel{n} = -\frac{25}{4} \end{cases} \Rightarrow -\frac{5}{2}m - m = 1 - \frac{25}{4} \to -\frac{7}{2}m = -\frac{21}{4} \to m = \frac{-21}{4} \times \frac{2}{-7}$$

$g(x) = f(x)$

$\frac{x-b}{2x^2-3x-5} = \frac{ax+2}{x^2-\frac{3}{2}x-\frac{5}{2}}$ ÷۲

$-m-n=1$

$-\frac{3}{2} - 1 = n \to \boxed{-\frac{5}{2} = n}$

$\boxed{m = \frac{3}{2}}$

$\frac{x-b}{2} = ax+2 \to \frac{x}{2} - \frac{b}{2} = ax+2 \to \frac{1}{2}x - \frac{1}{2}b = ax+2$

$am \cdot bn \to \left(\frac{1}{2} \times \frac{3}{2}\right) - \left(-4 \times -\frac{5}{2}\right) =$ $\boxed{a = \frac{1}{2}}$ $2 = -\frac{1}{2}b \to \boxed{4 = -b}$ $\boxed{b=-4}$

$\frac{3}{4} - 10 = \frac{3-40}{4} = \boxed{\frac{-37}{4}}$

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Subject:
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۶

$f(x)=\dfrac{bx+2}{8x+b}$ ، $8x+b=0 \to x=\dfrac{-b}{8}$

$D_f=\mathbb{R}-\dfrac{-b}{8}$

$D_f=D_g$

$\dfrac{-b}{8}=a$

$f(x)=g(x) \to \dfrac{bx+2}{8x+b}=c \to 8cx+bc=bx+2$

$8cx=bx$

$\boxed{8c=b}$ ، $\boxed{bc=2}$ $\Rightarrow \dfrac{bc=2}{8c} \to 8c^2=2 \to c^2=\dfrac{1}{4}$

$\boxed{c=\pm\dfrac{1}{2}}$

$b= \to 8\times\dfrac{1}{2}=\boxed{4}$ ، $\to 8\times -\dfrac{1}{2}=\boxed{-4}$

$a \to \dfrac{-\frac{1}{2}}{8}=\boxed{a=-\dfrac{1}{16}}$ ، $\to \dfrac{\frac{1}{2}}{8}=\boxed{a=\dfrac{1}{16}}$

$\dfrac{ab}{c}=\dfrac{\frac{1}{4}}{\frac{1}{2}}=\boxed{\pm\dfrac{2}{4}}$

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$2f-1=\{(-1,3),(2,7),(3,-5),(0,0)\} \to 2f=\{(-1,4),(2,8),(3,-4),(0,1)\}$

$f=\{(-1,2),(2,4),(3,-2),(0,\frac{1}{2})\}$ ، $g=\{(2,6),(3,6),(-1,-6),(5,0)\}$

$2g=\{(2,8),(3,12),(-1,-8)\}$ ، $f+g=\{(2,8),(3,4),(-1,-2)\}$

$\dfrac{2g}{f+g}=\{(2,1),(3,3),(-1,4)\}$

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$f=\{(2,a),(-3,3a-2b),(c,5),(2,d)\}$

$g=\{(2,-1),(-3,1),(a-3b,5)\}$

$\boxed{a=d=-1}$

$3a-2b=1 \to -3-1=2b$

$-6=2b \to \boxed{b=-2}$ (؟)

$\boxed{d+c=-1+5=4}$

$a-3b=c$

$-1+6=c \to \boxed{c=5}$

۹

$D_f=a$ $R_f=b$

$f(x)=\sqrt{-x^2+x-m} \to f(a)=\sqrt{-a^2+a-m}=b$

Arman

$F(x) = g(x)$

$$\frac{2}{x-c} = \frac{2x+a}{x^2+4x+b} \rightarrow \cancel{2x^2} + \underbrace{12x} + 2b = \cancel{2x^2} + \underbrace{xa - 2cx} - ca$$

$$12x = xa - 2cx \rightarrow 12 = a - 2c \rightarrow a = 12 - 2c$$

$$2b = -ca \rightarrow b = \frac{-ac}{2} \rightarrow$$

$$b = \frac{-12c + 2c^2}{2} , b = -4c + c^2$$

$x = c \rightarrow x - c = 0$

$$x = c \rightarrow x^2 + 4x + b = 0 \rightarrow c^2 + 4c + b = 0 \rightarrow c^2 + \cancel{4c} - \cancel{4c} + c^2 = 0$$

$$2c^2 = 0 \rightarrow c^2 = 0$$

$$\boxed{c = 0}$$

$$b = -4c + c^2 \rightarrow \boxed{b = 0}$$

$$a = 12 - \underset{0}{2c} \rightarrow \boxed{a = 12}$$

$$\underset{12}{a} + \underset{0}{b} + \underset{0}{c} = 12$$