15,50 زهراسادات حسینی

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۱) ۵

الف) $f(x) = \sqrt{x|x|}$, $g(x) = x$

if $x = -2$ $f(-2) =$ تعریف نشده ، $g(-2) = -2$ برابر نیست ، ✗

ب) $f(x) = \frac{\overbrace{(x+3)(x+1)}^{x^2+4x+3}+x+4}{x^2+5x+7}$, $g(x) = 1$ $f(x) = \frac{x^2+5x+7}{x^2+5x+7} = 1 \Rightarrow$

مساوی هستند ✓ $f(x) = 1$

ج) $f(x) = \frac{4\sin^2 x - 6\sin x}{2\sin x - 3}$, $g(x) = 2\sin x$

$f(x) = \frac{2\sin x \cancel{(2\sin x - 3)}}{\cancel{(2\sin x - 3)}} = 2\sin x$ برابرند ✓

د) $f(x) = \frac{x}{|x|}$, $g(x) = \frac{|x|}{x}$

$f(-2) \rightarrow \frac{-2}{2} = -1$ $g(-2) = \frac{2}{-2} = \frac{-2}{2} = -1$ برابرند ✓

$D_f = \mathbb{R} - \{0\}$ $D_g = \mathbb{R} - \{0\}$

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الف) $f(x) = \left[\frac{x^2}{x^2+1}\right]$, $g(x) = [x - [x]]$

$D_f = \mathbb{R}$ $D_g = \mathbb{R}$ $x \in \mathbb{Z} \rightarrow x - [x] = 0$

$x \notin \mathbb{Z} \rightarrow x - [x] = 0$

$x^2 < x^2 + 1$ $g(x) \underset{\text{همواره}}{=} 0$

$\left[\frac{x^2}{x^2+1}\right] = 0$

$f(x) \underset{\text{همواره}}{=} 0$ برابرند ✓

Arman

زهرا سادات حسینی

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ب) $f(x) = \frac{1}{[2x]}$, $g(x) = \frac{1}{2[x]}$

$x = \frac{3}{4}$

$\to f(x) = \frac{1}{[2 \times \frac{3}{4}]} = \frac{1}{[\frac{3}{2}]} = \frac{1}{1} = 1$

برابر نیستند ✗

$\to g(x) = \frac{1}{2[\frac{3}{4}]} = \frac{1}{2 \times 0} = \frac{1}{0}$ تعریف نشده

ج) $f(x) = \frac{1}{\sqrt{x - |x|}}$, $g(x) = \frac{1}{\sqrt{|x| - x}}$

$D_f = \mathbb{R}^+$ $D_g = \mathbb{R}^-$ برابر نیستند ✗

$f(-2) = \frac{1}{\sqrt{(-2) - |2|}} = \frac{1}{\sqrt{-4}}$ = تعریف نشده $g(-2) = \frac{1}{\sqrt{|-2| + 2}} = \frac{1}{2}$

د) $f(x) = \begin{cases} \frac{x^3 - x}{x-1} & ; x \neq 1 \\ 2 & ; x = 1 \end{cases}$, $g(x) = x^2 + x$

$\frac{x^3 - x}{x-1} = \frac{x(x^2-1)}{(x-1)} = \frac{x\cancel{(x-1)}(x+1)}{\cancel{(x-1)}} = x(x+1)$

برابرند ✓

$g(x) = x(x+1)$ $f(1) = 2$

$g(1) = 1 + 1 = 2$

۳)

$\begin{cases} f(x) + \cancel{g(x)} = x^2 + \cancel{x} + 1 \\ f(x) - \cancel{g(x)} = x^2 - \cancel{x} + 1 \end{cases} \Rightarrow 2f(x) = 2x^2 + 2 \to \boxed{f(x) = x^2 + 1}$

$\cancel{x^2} + \cancel{1} + g(x) = \cancel{x^2} + x + \cancel{1} \to \boxed{g(x) = x}$

$f^2(x) - g^2(x) \to (x^2+1)^2 - (x)^2 = x^4 + 1 + 2x^2 - x^2$

$\boxed{x^4 + x^2 + 1}$

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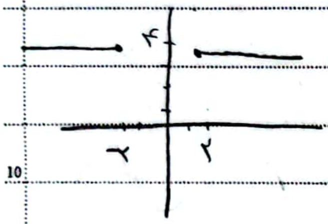
$F(x) = x - \sqrt{x^2-4} \to x - \overbrace{\sqrt{x^2-4}}^{a}$

$g(x) = x + \sqrt{x^2-4} \to x + \underbrace{\sqrt{x^2-4}}_{a}$

$F \times g = (x-a)(x+a) = x^2 - a^2$

$x^2 - \left(\sqrt{x^2-4}\right)^2 \to x^2 - (x^2-4) = x^2 - x^2 + 4 = \boxed{4}$



$D_F = \sqrt{x^2-4} = \sqrt{(x-2)(x+2)}$ → 2 , -2

	-2		2	
$+$	ϕ	$-$	ϕ	$+$

x باید متعلق به دامنه باشد

$\boxed{D_g = D_F = (-\infty, -2] \cup [2, +\infty) \qquad F \times g = 4}$

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$F(x) = g(x)$

$g(x) = \dfrac{x-b}{2x^2-3x-5} \to 2x^2-3x-5=0 \to x^2-3x-10=0 \to (x+2)(x-5)=0$

ریشه‌ها $\dfrac{5}{2}$ ، $\dfrac{-2}{2} = -1$

ریشه‌های $g(x)$ را پیدا می‌کنیم و آن عددها باید ریشه‌ی $F(x)$ هم باشند.

$F(x) = \dfrac{ax+2}{x^2-mx+n} \to x^2 - mx + n \xrightarrow{\boxed{x=-1}} (1 + m + n = 0) \times(-1)$

$x^2 - mx + n \xrightarrow{\boxed{x=\frac{5}{2}}} \dfrac{25}{4} - \dfrac{5}{2}m + n = 0$

$\begin{cases} -m - \cancel{n} = 1 \\ -\dfrac{5}{2}m + \cancel{n} = -\dfrac{25}{4} \end{cases} \Rightarrow -\dfrac{5}{2}m - m = 1 - \dfrac{25}{4} \to -\dfrac{7}{2}m = -\dfrac{21}{4} \to m = \dfrac{\cancel{42}^{3}}{\cancel{28}_{2}}$

$\boxed{m = \dfrac{3}{2}}$

$-m - n = 1$

$-\dfrac{3}{2} - 1 = n \to \boxed{-\dfrac{5}{2} = n}$

$g(x) = F(x)$

$\dfrac{x-b}{2x^2-3x-5} = \dfrac{ax+2}{x^2 - \frac{3}{2}x - \frac{5}{2}}$ $\div 2$

$\dfrac{x-b}{2} = ax+2 \to \dfrac{x}{2} - \dfrac{b}{2} = ax+2 \to \dfrac{1}{2}x - \dfrac{1}{2}b = ax+2$

$\boxed{a = \dfrac{1}{2}}$ $\quad 2 = -\dfrac{1}{2}b \to \boxed{4 = -b}$ $\quad \boxed{b = -4}$

$am \cdot bn \to \left(\dfrac{1}{2} \times \dfrac{3}{2}\right) - \left(-4 \times -\dfrac{5}{2}\right) = \dfrac{3}{4} - 10 = \dfrac{3-40}{4} = \boxed{-\dfrac{37}{4}}$

6 — 1

$f(x) = \frac{bx+2}{8x+b}$; $8x+b=0 \to x = \frac{-b}{8}$

$D_f = \mathbb{R} - \frac{-b}{8}$

$D_f = D_g$

$\frac{-b}{8} = a$

$f(x) = g(x) \to \frac{bx+2}{8x+b} = c \to 8cx + bc = bx + 2$

$8cx = bx$

$\boxed{8c = b}$

$\boxed{bc = 2}$ $\to \frac{bc = 2}{8c} \to 8c^2 = 2 \to c^2 = \frac{1}{4}$

$\boxed{c = \pm\frac{1}{2}}$ ✓

$b = \to 8 \times \frac{1}{2} = \boxed{4}$ ✓

$\to 8 \times -\frac{1}{2} = \boxed{-4}$ ✓

$a \to \frac{-\frac{1}{2}}{8} = \cancel{a = \frac{1}{16}}$

$\to \frac{\frac{1}{2}}{8} = \cancel{a = \frac{1}{16}}$

$\frac{ab}{c} = \frac{\frac{1}{4}}{\frac{1}{2}} = \cancel{\pm\frac{2}{4}}$

3

7

$2f - 1 = \{(-1,3),(2,7),(3,-5),(0,0)\} \to 2f = \{(-1,4),(2,8),(3,-4),(0,1)\}$

$f = \{(-1,2),(2,4),(3,-2),(0,\frac{1}{2})\}$ $\quad$ $g = \{(2,4),(3,6),(-1,-4),(5,0)\}$

$2g = \{(2,8),(3,12),(-1,-8)\}$ $\quad$ $f+g = \{(2,8),(3,4),(-1,-2)\}$

$\frac{2g}{f+g} = \{(2,1),(3,3),(-1,4)\}$

2

8

$f = \{(2,a),(-3,3a-2b),(c,5),(2,d)\}$

$g = \{(2,-1),(-3,1),(a-3b,5)\}$

$\boxed{a = d = -1}$

$3a - 2b = 1 \to -3 - 1 = 2b$

$-4 = 2b \to \boxed{b = -2}$

$a - 3b = c$

$-1 + 6 = c \to \boxed{c = 5}$

$\boxed{d + c = -1 + 5 = 4}$

9 — 5

$D_f = a \quad R_f = b$

$f(x) = \sqrt{-x^2 + x - m} \to f(a) = \sqrt{-a^2 + a - m} = b$

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کامل

$F(x) = g(x)$

$\frac{2}{x-c} = \frac{2x+a}{x^2+6x+b} \rightarrow \underbrace{2x^2 + \underbrace{12x}_{} + 2b = 2x^2 + \underbrace{xa - 2cx}_{}}_{} - ca$

$12x = xa - 2cx \rightarrow 12 = a - 2c \rightarrow a = 12 - 2c$

$2b = -ca \rightarrow b = \frac{-ac}{2} \rightarrow$

$b = \frac{-12c + 2c^2}{2} , b = -6c + c^2$

$x = c \rightarrow x - c = 0$

$x = c \rightarrow x^2 + 6x + b = 0 \rightarrow c^2 + 6c + b = 0 \rightarrow c^2 + \cancel{6c} - \cancel{6c} + c^2 = 0$

$2c^2 = 0 \rightarrow c^2 = 0 \rightarrow \boxed{c = 0}$

$b = -6c + c^2 \rightarrow \boxed{b = 0}$

$a = 12 - \underset{0}{2c} \rightarrow \boxed{a = 12}$

$\underset{12}{a} + \underset{0}{b} + \underset{0}{c} = 12$