Subject هندسه تحلیلی - دهم ریاضی - تمرین شماره ۵

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① الف) $f(x)=\sqrt{x|x|}$, $g(x)=x$

$x|x| \geq 0 \rightarrow x \in [0,+\infty)$ ↳ $x \in \mathbb{R}$ مساوی نیست.

ب) $f(x)=\frac{(x+2)^3(x+1)+x+6}{x^2+5x+7}$, $g(x)=1$

$x^2+5x+7 \rightarrow \Delta = b^2-4ac = 5^2-4\times1\times7 = 25-28 = -3$ بیشتر باید

↳ $\Delta<0$

$\rightarrow D_f=\mathbb{R}$ $D_g=\mathbb{R}$ $\rightarrow D_f=D_g$ مساوی است.

ج) $f(x)=\frac{4\sin^2 x-6\sin x}{2\sin x-3}$, $g(x)=2\sin x$

$2\sin x-3\neq 0 \rightarrow 2\sin x \neq 3 \rightarrow \sin x \neq \frac{3}{2}$ همواره برقرار است

$D_f=\mathbb{R}$ $D_g=\mathbb{R}$ مساوی است

د) $f(x)=\frac{x}{|x|}$, $g(x)=\frac{|x|}{x}$

$|x|\neq 0 \rightarrow x\neq 0$ $D_f=D_g=\mathbb{R}-\{0\}$ مساوی است

② الف) $f(x)=\left[\frac{x^2}{x^2+1}\right]$, $g(x)=[x-[x]]$

$f(x)=0$ ↳ $x-[x]=[0,1) \rightarrow [x-[x]]=0$

مساوی است.

ب) $f(x)=\frac{1}{[2x]}$, $g(x)=\frac{1}{2[x]}$ مساوی نیست

$\mathbb{R}-[0,\frac{1}{2})$ ↳ $\mathbb{R}-[0,1)$

ج) $fx=\frac{1}{\sqrt{x-|x|}}$, $g(x)=\frac{1}{\sqrt{|x|-x}}$

$x-|x|>0 \rightarrow x>|x| \rightarrow D_f=\emptyset$ مساوی نیست

$|x|-x>0 \rightarrow |x|>x \rightarrow D_g=(-\infty,0)$

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۱) $f(x)=\begin{cases}\frac{x^3-x}{x-1} & ;x\neq 1\\ 2 & ;x=1\end{cases}$, $g(x)=x^2+x$

$\frac{x^3-x}{x-1}=\frac{x(x^2-1)}{x-1}=\frac{x(x+1)(x-1)}{x-1}=x(x+1)=x^2+x$

همواری دارد.

۲) $f(x)+g(x)=x^2+x+1$

$f(x)-g(x)=x^2-x+1$

$f^2(x)-g^2(x)$

$f(x)+g(x)=x^2+x+1$

$+\ f(x)-g(x)=x^2-x+1$

$2f(x)=2x^2+2 \rightarrow f(x)=x^2+1 \rightarrow \boxed{f(x)=x^2+1}$

$f(x)+g(x)=x^2+x+1 \rightarrow x^2+1+g(x)=x^2+x+1$

$\boxed{g(x)=x}$

$f^2(x)-g^2(x)=(x^2+1)^2-x^2=x^4+2x^2+1-x^2=$

$\boxed{f^2(x)-g^2(x)=x^4+x^2+1}$

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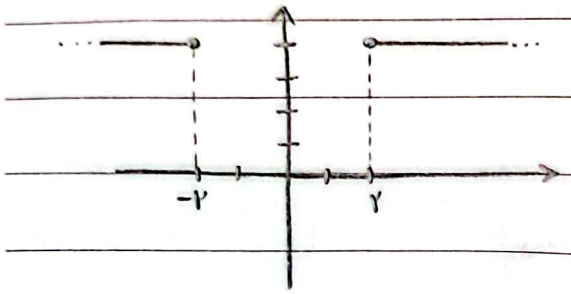
④ $f(x) = x - \sqrt{x^2 - 4}$ $\quad x^2 - 4 \geq 0 \rightarrow x^2 \geq 4$

$g(x) = x + \sqrt{x^2 - 4}$ $\quad x \geq 2 \ , \ x \leq -2$

$f.g =$

$(x - \sqrt{x^2-4})(x + \sqrt{x^2-4}) = x^2 - (x^2-4) = 4$



⑤ $f(x) = \dfrac{ax+2}{x^2 - mx + n}$ $\quad g(x) = \dfrac{x-b}{x^2 - 4x + 4}$ $\quad am - bn = ?$

$x^2 - 4x + 4 = 0 \rightarrow \Delta = b^2 - 4ac = (-4)^2 - 4 \times 1 \times 4 = 16 - 16 = 0$

$x = \dfrac{-b \pm \sqrt{\Delta}}{2a} = \dfrac{4 \pm 0}{2} \rightarrow x = 2$

$x^2 - mx + n = 0 \rightarrow \Delta = b^2 - 4ac = (-m)^2 - 4 \times 1 \times n = m^2 - 4n$

$\dfrac{-b \pm \sqrt{\Delta}}{2a} = \dfrac{m \pm \sqrt{m^2 - 4n}}{2} = \dfrac{m \pm \sqrt{4m^2 - 16n}}{4} = \dfrac{m \pm 4}{4}$

$m \pm 4 = \sqrt{4m^2 - 16n} \rightarrow m^2 \pm 8m + 16 = 4m^2 - 16n$

$\sqrt{4m^2 - 16n} = 4 \rightarrow 4m^2 - 16n = 16 \rightarrow m^2 - 4n = 4$

$\rightarrow m^2 - 4n = 4 \rightarrow 4n = m^2 - 4 \rightarrow n = \dfrac{m^2-4}{4}$

$-4n = 0 \rightarrow n = 0 \rightarrow m = \pm 2$

$\dfrac{ax+2}{x^2 - mx + n} = \dfrac{x-b}{x^2 - 4x + 4} \rightarrow \dfrac{ax+2}{(x-2)(x-2)} = \dfrac{x-b}{(x-2)^2}$

$ax + 2 = x - b \rightarrow ax + 2 = x - b$

$a = 1 \rightarrow a = 1$

$2 = -b \rightarrow b = -2$

$am - bn = 1 \times 2 - (-2) \times 0 = 2$

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$f(x)=\frac{bx+2}{8x+b}$ $\quad g(x)=c$ (9)

$D_g=\mathbb{R}-\{a\}$

$\frac{ab}{c}=\frac{-c\times 8c}{c}=\frac{-8c^2}{c}=-8c$

$8a+b=0 \rightarrow 8a=-b \rightarrow a=\frac{-b}{8}=-\frac{b}{8}=-\frac{8c}{8}=-c \rightarrow \boxed{a=-c}$

$f(1)=\frac{b+2}{8+b}=c \rightarrow b+2=8c+bc$

$f(2)=\frac{2b+2}{14+b}=c \rightarrow 2b+2=14c+bc$

$-b-2=-8c-bc$

$\boxed{b=8c}$

$b+2=8c+bc \rightarrow 8c+2=8c+8c^2 \rightarrow 8c^2=2 \rightarrow c^2=\frac{2}{8}$

$c^2=\frac{2}{8}=\frac{1}{4} \rightarrow c=\pm\frac{1}{2}$

$\frac{ab}{c}=-8c=-8(\pm\frac{1}{2})=\boxed{\mp 4}$

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$2f-1=\{(-1,3),(2,7),(3,-5),(0,0)\}$ (۷)

$g=\{(2,2),(3,4),(-1,-2),(5,0)\}$

$2g=\{(2,8),(3,11),(-1,-8),(5,0)\}$

$f=\{(-1,2),(2,5),(3,-2),(0,\frac{1}{2})\}$

$f+g=\{(2,8),(3,4),(-1,-2)\}$

$R=\{(2,1),(3,3),(-1,5)\}$

$f=\{(2,a),(-3,3a-2b),(c,5),(2,d)\}$ $d+c$ (۸)

$g=\{(2,-1),(-3,1),(a-3b,5)\}$

$a=d=-1$

$3a-2b=1 \rightarrow 3(-1)-2b=1 \rightarrow -3-2b=1 \rightarrow -3-1=2b \rightarrow -4=2b \rightarrow b=-2$

$c=a-3b=-1-3(-2)=-1+6=+5 \rightarrow c=+5$

$d+c=-1+5=+4 \rightarrow \boxed{d+c=4}$

$f(x)=\sqrt{-x^2+x-m}$ (۹)

$g(x)=\{(a,b)\}$

$-x^2+x-m \geq 0 \rightarrow \Delta=b^2-4ac=1^2-4(-1)(-m)=1-4m=0 \rightarrow 4m=1 \rightarrow$

$4m=1 \rightarrow m=\frac{1}{4}$

$-x^2+x-\frac{1}{4} \rightarrow -(x^2-x+\frac{1}{4})=\sqrt{-(x-\frac{1}{2})^2} \rightarrow (x-\frac{1}{2})^2=0 \rightarrow x-\frac{1}{2}=0$

$x=\frac{1}{2}$ $f(x)=\sqrt{-\frac{1}{4}+\frac{1}{2}-\frac{1}{4}}=\sqrt{0}=0 \rightarrow g(x)=\{(\frac{1}{2},0)\}$

$a+b=\frac{1}{2}+0=\boxed{\frac{1}{2}}$

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① $f(x)=\frac{rx+a}{x^r+qx+b}$ $\quad g(x)=\frac{r}{x-c}$

$x-c\neq 0 \rightarrow x\neq c$

$x^r+qx+b=(x-c)^r$

$x^r+qx+b=x^r-rxc+c^r$

$q=-rc \rightarrow c=-r^0$

$b=c^0 \rightarrow b=(-r^0)^r=q$

$b=q \quad c=-r^0$

$\frac{rx+a}{x^r+qx+q}=\frac{r}{x+r^0} \rightarrow \frac{rx+a}{(x+r^0)^r}=\frac{r}{x+r^0} \xrightarrow{\times(x+r^0)^r} rx+a=r(x+r^0)$

$rx+a=rx+q \rightarrow a=q$

$a+b+c=q+q-r^0=1r$

$\boxed{a+b+c=1r}$