

الف) $y = \frac{x+4}{x^3-20x^2+31x-15}$ $\frac{x^3-20x^2+31x-15}{x^3-20x^2+31x-15} = \frac{x-1}{x^2-19x+15} = (x-1)(x-5)$
 $\frac{x^3-20x^2+31x-15}{x^3-20x^2+31x-15} = \frac{x-1}{x^2-19x+15} \rightarrow (x-1)(x-5)(x-1) \neq 0 \rightarrow D_f = \mathbb{R} - \{1, 5, \frac{15}{x}\}$
 ب) $y = \frac{x+4}{x^3-20x^2+31x-15}$ $\frac{x^3-20x^2+31x-15}{x^3-20x^2+31x-15} = \frac{x-1}{x^2-19x+15} \rightarrow x+1 \neq 0 \rightarrow x \neq -1$
 $\frac{x^3-20x^2+31x-15}{x^3-20x^2+31x-15} = \frac{x-1}{x^2-19x+15} \rightarrow \frac{x^3-20x^2+31x-15}{x^3-20x^2+31x-15} = \frac{x-1}{x^2-19x+15}$
 $\frac{x^3-20x^2+31x-15}{x^3-20x^2+31x-15} = \frac{x-1}{x^2-19x+15} \rightarrow \frac{x^3-20x^2+31x-15}{x^3-20x^2+31x-15} = \frac{x-1}{x^2-19x+15}$
 $D_f = \mathbb{R} - \{2, \frac{1}{x}, -13\}$

الف) $y = \frac{x+1}{x-\sqrt{x^2-2x}}$ $\frac{x+1}{x-\sqrt{x^2-2x}} = \frac{x+1}{x-\sqrt{x^2-2x}} \rightarrow x+1 \neq 0 \rightarrow x \neq -1$
 $\frac{x+1}{x-\sqrt{x^2-2x}} = \frac{x+1}{x-\sqrt{x^2-2x}} \rightarrow x+1 \neq 0 \rightarrow x \neq -1$
 $D_f = (-\infty, \frac{1}{x}] - \{13\}$
 ب) $y = \frac{x+1}{x-\sqrt{x^2-2x}}$ $\frac{x+1}{x-\sqrt{x^2-2x}} = \frac{x+1}{x-\sqrt{x^2-2x}} \rightarrow x+1 \neq 0 \rightarrow x \neq -1$
 $\frac{x+1}{x-\sqrt{x^2-2x}} = \frac{x+1}{x-\sqrt{x^2-2x}} \rightarrow x+1 \neq 0 \rightarrow x \neq -1$
 $D_f = [\frac{1}{x}, +\infty) - \{1, 23\}$

الف) $y = \frac{\sin x}{\cos x-1}$ $\frac{\sin x}{\cos x-1} = \frac{\sin x}{\cos x-1} \rightarrow \cos x \neq 1 \rightarrow \cos x \neq \frac{1}{x}$
 $\frac{\sin x}{\cos x-1} = \frac{\sin x}{\cos x-1} \rightarrow \cos x \neq 1 \rightarrow \cos x \neq \frac{1}{x}$
 $D_f = \mathbb{R} - \{2k\pi + \frac{\pi}{x}, 2k\pi + \frac{5\pi}{x}\}$
 ب) $y = \frac{\cos x+1}{\sin x+1}$ $\frac{\cos x+1}{\sin x+1} = \frac{\cos x+1}{\sin x+1} \rightarrow \sin x \neq -\frac{1}{x}$
 $\frac{\cos x+1}{\sin x+1} = \frac{\cos x+1}{\sin x+1} \rightarrow \sin x \neq -\frac{1}{x}$
 $D_f = \mathbb{R} - \{2k\pi + \frac{\pi}{x}, 2k\pi + \frac{11\pi}{x}\}$
 ج) $y = \frac{\tan x+1}{\cot x-1}$ $\frac{\tan x+1}{\cot x-1} = \frac{\tan x+1}{\cot x-1} \rightarrow \cot x \neq 1 \rightarrow \cot x \neq \frac{1}{x}$
 $\frac{\tan x+1}{\cot x-1} = \frac{\tan x+1}{\cot x-1} \rightarrow \cot x \neq 1 \rightarrow \cot x \neq \frac{1}{x}$
 $D_f = \mathbb{R} - \{k\pi, k\pi + \frac{\pi}{x}, k\pi + \frac{3\pi}{x}\}$
 د) $y = \frac{\sin x+1}{\cos x-1}$ $\frac{\sin x+1}{\cos x-1} = \frac{\sin x+1}{\cos x-1} \rightarrow \sin x \neq \frac{1}{x} \rightarrow \sin x \neq \pm \frac{\sqrt{x}}{x}$
 $\frac{\sin x+1}{\cos x-1} = \frac{\sin x+1}{\cos x-1} \rightarrow \sin x \neq \frac{1}{x} \rightarrow \sin x \neq \pm \frac{\sqrt{x}}{x}$
 $D_f = \mathbb{R} - \{k\pi \pm \frac{\pi}{x}\}$

الف) $x^2-5x+4 > 0 \rightarrow (x-2)(x-3) > 0$ $\frac{1}{x-2} - \frac{1}{x-3} > 0$ $(3, +\infty) \cup (-\infty, 2)$
 ب) $x^2-4x+5 < 0 \rightarrow (x-1)(x-5) < 0$ $\frac{1}{x-1} - \frac{1}{x-5} < 0$ $(1, 5)$
 ج) $x^2+4x+5 > 0 \rightarrow (x+1)(x+5) > 0$ $\frac{-5}{x+1} - \frac{-1}{x+5} > 0$ $[-1, +\infty) \cup (-\infty, -5]$
 د) $x^2-7x+4 < 0 \rightarrow (x-1)(x-4) < 0$ $\frac{1}{x-1} - \frac{1}{x-4} < 0$ $(1, 4)$

الف) $\frac{x^2-3x+2}{x-5} < 0$ $\frac{(x-1)(x-2)}{x-5} < 0$ $\frac{1}{x-1} - \frac{1}{x-2} < \frac{1}{x-5}$ $(-\infty, 1) \cup (2, 5)$
 ب) $\frac{x^2-1}{x^2-4x+5} > 0$ $\frac{(x-1)(x+1)}{(x-1)(x-5)} > 0$ $\frac{-1}{x-1} - \frac{1}{x-5} > 0$ $(5, +\infty) \cup (-\infty, -1]$

$$\text{ا) } y = \sqrt{\frac{x^2 - 4x + 3}{x^2 - 1}} \quad \frac{(x-1)(x-3)}{(x-1)(x^2+1)} > 0$$

$$\hookrightarrow x^2 - 1 \neq 0 \rightarrow x \neq \pm 1 \rightarrow x \neq \pm 1 \quad \hookrightarrow \Delta = b^2 - 4ac = 16 - 12 = 4 \rightarrow x = 2 \text{ و } x = -2$$

$$D_f = [2, +\infty)$$

$$\text{ب) } y = \sqrt{\frac{x^2 - 4x + 3}{x^2 - 1}} \rightarrow x^2 - 1 \neq 0 \rightarrow x \neq \pm 1 \rightarrow x \neq \pm 1 \quad \hookrightarrow D_f = \mathbb{R} - \{\pm 1\}$$

$$\text{ا) } y = \log(x^2 - 3x) \quad x^2 - 3x > 0 \rightarrow x(x-3) > 0 \quad D_f = (-\infty, 0) \cup (3, +\infty)$$

$$\text{ب) } y = \log \frac{14-x^2}{|x|-1} \quad 14-x^2 > 0 \rightarrow x^2 < 14 \rightarrow -\sqrt{14} < x < \sqrt{14}$$

$$|x|-1 > 0 \rightarrow |x| > 1 \rightarrow x > 1 \text{ و } x < -1 \quad \hookrightarrow x \neq \pm 1 \rightarrow x \neq \pm 1$$

$$D_f = (-\sqrt{14}, -1) \cup (1, \sqrt{14}) - \{\pm 1\}$$

$$\text{ج) } y = \log \frac{x^2 - 4x + 3}{x-1} \quad \frac{(x-3)(x-1)}{x-1} > 0 \quad \hookrightarrow x-3 > 0 \text{ و } x-1 > 0 \rightarrow x > 3$$

$$D_f = (3, +\infty)$$

$$\text{د) } y = \sqrt{x^2 - 11x + 18} + \log \frac{14-x^2}{|x|-1} \quad x^2 - 11x + 18 > 0 \rightarrow (x-2)(x-9) > 0 \rightarrow x < 2 \text{ و } x > 9$$

$$14-x^2 > 0 \rightarrow x^2 < 14 \rightarrow -\sqrt{14} < x < \sqrt{14}$$

$$|x|-1 > 0 \rightarrow |x| > 1 \rightarrow x > 1 \text{ و } x < -1$$

$$D_f = (0, 2) \cup (9, +\infty)$$

$$y = \frac{x^2 - x}{x^2 + x} = \frac{x(x-1)}{x(x+1)}$$

x	-1	0	1
y	+	-	+

x	-1	0	1
y	+	-	+

x	-1	0	1
y	+	-	+

$$y = \sqrt{\frac{x-5}{x}}$$

x	-1	0	1	2	3
y	+	-	0	+	+

$$D_f = (-2, -1] \cup (2, 3]$$

