

المعادلة الأصلية: $\frac{x^2 + 1}{x^2 - 1} = \frac{x + 1}{x - 1}$

$$y = \frac{x+1}{x^2-1} = \frac{x+1}{(x-1)(x+1)}$$

$$\downarrow$$

$$\frac{1}{x-1}$$

$$D_f = \mathbb{R} - \left\{ -1, \frac{1}{x-1} \right\}$$

$$y = \frac{x+1}{x^2-1} = \frac{x+1}{(x-1)(x+1)}$$

$$\downarrow$$

$$\frac{1}{x-1}$$

$$D_f = \mathbb{R} - \left\{ 1, \frac{1}{x-1} \right\}$$

$$y = \frac{x+1}{x - \sqrt{x^2-1}}$$

$$x - \sqrt{x^2-1} > 0 \Rightarrow (x > \sqrt{x^2-1})$$

$$x^2 > x^2 - 1 \Rightarrow 1 > 0$$

$$D_f = \left[\frac{1}{x}, +\infty \right) - \left\{ 1, 2 \right\}$$

$$y = \frac{x+1}{x - \sqrt{x^2-1}}$$

$$x - \sqrt{x^2-1} > 0 \Rightarrow (x > \sqrt{x^2-1})$$

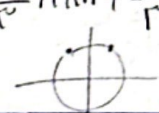
$$x^2 > x^2 - 1 \Rightarrow 1 > 0$$

$$D_f = \left[-\infty, \frac{1}{x} \right] - \left\{ 1 \right\}$$

$$y = \frac{\sin x + 1}{\sin^2 x - 1}$$

$$\sin^2 x - 1 \neq 0$$

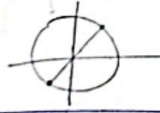
$$\sin x \neq \pm 1$$

$$D_f = \mathbb{R} - \left\{ k\pi + \frac{\pi}{2}, k\pi + \frac{3\pi}{2} \right\}$$


$$y = \frac{\tan x + 1}{\cot x - 1}$$

$$\cot x - 1 \neq 0$$

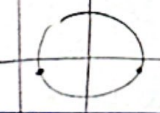
$$\cot x \neq 1$$

$$D_f = \mathbb{R} - \left\{ k\pi - \frac{\pi}{4} \right\}$$


$$y = \frac{\cos x - 1}{\sin x + 1}$$

$$\sin x + 1 \neq 0$$

$$\sin x \neq -1$$

$$D_f = \mathbb{R} - \left\{ k\pi + \frac{3\pi}{2} \right\}$$


$$y = \frac{\sin x}{\cos x - 1}$$

$$\cos x - 1 \neq 0$$

$$\cos x \neq 1$$

$$D_f = \mathbb{R} - \left\{ k\pi, k\pi + \frac{\pi}{2} \right\}$$


$$x^2 - 4x + 4 \leq 0$$

$$(x-2)^2 \leq 0$$

$$x = 2$$

$$x^2 + 4x + 8 > 0$$

$$(x+2)^2 + 4 > 0$$

$$x \in \mathbb{R}$$

$$x^2 - 4x + 8 < 0$$

$$(x-2)^2 + 4 < 0$$

$$\text{No solution}$$

$$x^2 - 8x + 4 > 0$$

$$(x-4)^2 - 12 > 0$$

$$x < 4 - 2\sqrt{3} \text{ or } x > 4 + 2\sqrt{3}$$

$$\frac{x^2-1}{x^2-4x+8} > 0$$

$$\frac{(x-1)(x+1)}{(x-4)^2+4} > 0$$

$$D_f = (-\infty, -1] \cup (\delta, +\infty)$$

$$\frac{x^2-1}{x^2-4x+8} < 0$$

$$\frac{(x-1)(x+1)}{(x-4)^2+4} < 0$$

$$D_f = (-\infty, 1) \cup (2, \delta)$$

$$y = \left| \frac{n^2 - \varepsilon n + 1}{n^2 - 1} \right| \Rightarrow \frac{n^2 - \varepsilon n + 1}{n^2 - 1} \Rightarrow$$

$$n^2 - 1 \neq 0 \quad \underbrace{(n+1)}_{-1} \underbrace{(n-1)}_{+1} \neq 0$$

$$R_f: \mathbb{R} - \{-1, +1\}$$

$$y = \sqrt{\frac{(n^2 - \varepsilon n - 1)}{n^2 - 1}} \Rightarrow \sqrt{\frac{(n-1)(n+1)}{n^2 - 1}} \Rightarrow$$

$$\frac{(n-1)(n+1)}{n^2 - 1} \neq 0 \Rightarrow \frac{n^2 - 1}{n^2 - 1} \neq 0 \Rightarrow$$

$$\frac{n^2 - 1}{n^2 - 1} \neq 0 \Rightarrow \frac{n^2 - 1}{n^2 - 1} \neq 0 \Rightarrow$$

$$R_f: [1, +\infty)$$

$$\sqrt{n^2 - 11n + 18} + \log_m \sqrt{104 - n}$$

$$\frac{(n-4)(n-3)}{\varepsilon} \geq 0 \quad \frac{\varepsilon}{\varepsilon} \geq 0 \quad \frac{1}{\varepsilon} \geq 0$$

$$\sqrt{104 - n} \geq 0 \quad \frac{(4-n)(4+n)}{4} \geq 0 \quad \frac{(4-n)(4+n)}{4} \geq 0$$

$$\frac{-4}{-4} \frac{+4}{+4} \quad n \geq 0 \quad n \neq 1$$

$$R_f: (-4, \varepsilon] \cup [4, +\infty) - \{1\}$$

$$R_f: (\varepsilon, 1) - \{9\}$$

$$y = \log_m |4 - n|$$

$$|4 - n| > 0 \quad (\varepsilon - n)(4 + n) \geq 0 \quad n^2 - \varepsilon n \geq 0$$

$$\frac{-\varepsilon}{-4} \frac{\varepsilon}{+4} \frac{\varepsilon}{+4} \quad \frac{-\varepsilon}{-4} \frac{\varepsilon}{+4} \quad \frac{-\varepsilon}{-4} \frac{\varepsilon}{+4}$$

$$|4 - n| > 0 \quad |n| > 4$$

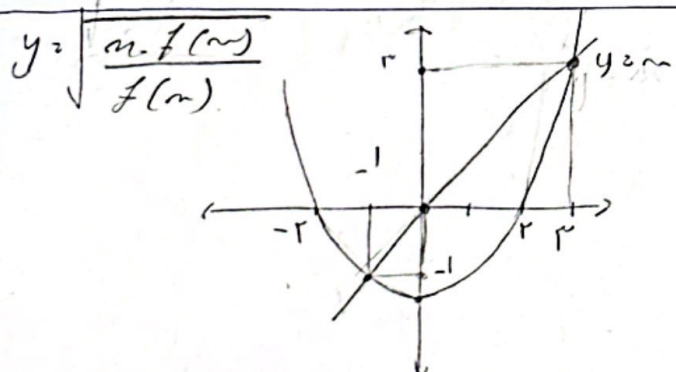
$$R_f: (-\varepsilon, \varepsilon) \cup [4, +\infty)$$

$$R_f: (-\infty, -4) \cup (4, +\infty)$$

$$y = \frac{n^2 - n}{n^2 + n} \Rightarrow \frac{n^2 - n}{n^2 + n} \Rightarrow \frac{n(n-1)}{n(n+1)}$$

$$\frac{-1}{+1} \frac{+1}{+1} \quad \frac{+1}{-1} \quad \frac{+1}{-1}$$

	-1	0	+1
$n^2 - n$	+	+	-
$n^2 + n$	+	-	+
$y(n)$	+	-	+



$$-r, r \in \mathbb{R}$$

$$\frac{-r}{+1} \quad \frac{r}{-1} \quad \frac{r}{+1}$$

$$R_f: [-2, -1] \cup [1, +\infty)$$

