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باران قاسمی

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الف) $y=\dfrac{x+4}{4x^3-20x^2+31x-15}=\dfrac{x+4}{(x-1)\underbrace{(4x^2-16x+15)}_{(2x-5)(2x-3)}}=(x-1)(2x-5)(2x-3)\neq 0 \Rightarrow D_f=\mathbb{R}-\{1,\frac{5}{2},\frac{3}{2}\}$

ب) $\dfrac{x+3}{3x^3-x^2-8x-4}=\dfrac{x+3}{(x+1)(3x^2-4x-4)} \Rightarrow D_f=\mathbb{R}-\{-1,2,-\frac{2}{3}\}$

$3x^2-4x-4=0 \to x^2-4x-12=0 \to (x-6)(x+2)=0$

$x=\frac{6}{3}=2 \ ,\ x=-\frac{2}{3}$

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الف) $y=\dfrac{x+1}{x-\sqrt{3-2x}}$

$3-2x\geq 0 \to 3\geq 2x \to \frac{3}{2}\geq x \Rightarrow D_f=(-\infty,\frac{3}{2}]-\{1\}$

$x-\sqrt{3-2x}\neq 0 \to x\neq\sqrt{3-2x} \to x^2\neq 3-2x$

$x^2+2x-3\neq 0$

$(x-1)(x+3)\neq 0 \to x\neq 1 \ ,\ x\neq -3$

ب) $y=\dfrac{x+2}{x-\sqrt{3x-2}}$

$3x-2\geq 0 \to 3x\geq 2 \to x\geq \frac{2}{3} \Rightarrow D_f=[\frac{2}{3},+\infty)-\{1,2\}$

$x-\sqrt{3x-2}\neq 0 \to x\neq\sqrt{3x-2} \to x^2\neq \cancel{3x-2} \to x^2-3x+2\neq 0$

$(x-2)(x-1)\neq 0 \to x\neq 2 \ ,\ x\neq 1$

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الف) $y=\dfrac{\sin x}{2\cos x-1}=2\cos x\neq 1 \Rightarrow \cos x\neq \frac{1}{2} \to D_f=\mathbb{R}-\{2k\pi\pm\frac{\pi}{3}\}$

ب) $y=\dfrac{\cos x+1}{2\sin x+1}=2\sin x\neq -1 \Rightarrow \sin x\neq -\frac{1}{2} \to D_f=\mathbb{R}-\{2k\pi-\frac{\pi}{6},\ 2k\pi+\pi+\frac{\pi}{6}\}$

ج) $y=\dfrac{\tan x+2}{\cot x-1}$

$\tan x\neq$ ت ن $\to \mathbb{R}-\{2k\pi\pm\frac{\pi}{2}\}$

$\cot x\neq$ ت ن $\to \mathbb{R}-\{k\pi\}$

$\cot x-1\neq 0 \to \cot x\neq 1 \to \mathbb{R}-\{k\pi+\frac{\pi}{4}\}$

$D_f=\mathbb{R}-\{\frac{k\pi}{2},\ k\pi+\frac{\pi}{4}\}$

د) $y=\dfrac{\sin x+1}{4\sin^2 x-3}=4\sin^2 x\neq 3 \to \sin^2 x\neq\frac{3}{4} \to \sin x\neq\pm\frac{\sqrt{3}}{2} \Rightarrow D_f=\mathbb{R}-\{k\pi\pm\frac{\pi}{3}\}$

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الف) $x^2-5x+6>0 \to (x-3)(x-2)>0$ $\quad \frac{2 \quad 3}{+ \; 0 \; - \; 0 \; +} \to D_f=(-\infty,2)\cup(3,+\infty)$

ب) $x^2-6x+5<0 \to (x-5)(x-1)$ $\quad \frac{1 \quad 5}{+ \; 0 \; - \; 0 \; +} \to D_f=(1,5)$

ج) $x^2+6x+5 \to (x+5)(x+1)\geq 0$ $\quad \frac{-5 \quad -1}{+ \; 0 \; - \; 0 \; +} \to D_f=(-\infty,-5]\cup[-1,+\infty)$

د) $x^2-7x+6\leq 0 \to (x-6)(x-1)\leq 0$ $\quad \frac{1 \quad 6}{+ \; 0 \; - \; 0 \; +} \to D_f=[1,6]$

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الف) $\frac{x^2-3x+2}{x-5}<0 \to \frac{(x-2)(x-1)}{x-5}<0$ $\quad \frac{1 \quad 2 \quad 5}{- \; 0 \; + \; 0 \; - \; \| \; +} \to D_f=(-\infty,1)\cup(2,5)$

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ب) $\frac{x^2-1}{x^2-6x+5}\geq 0 \to \frac{(x+1)(x-1)}{(x-5)(x-1)}$ $\quad \frac{-1 \quad 1^{*} \quad 5}{+ \; 0 \; - \; \| \; - \; \| \; +} \to D_f=(-\infty,-1]\cup(5,+\infty)$

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الف) $y=\sqrt{\frac{x^2-4x+3}{x^3-1}} \to \frac{(x-3)(x-1)}{(x-1)\underbrace{(x^2+x+1)}_{\text{همیشه } +}}\geq 0$ $\quad \frac{1^{*} \quad 3}{- \; \| \; - \; 0 \; +} \to D_f=[3,+\infty)$

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ب) $\sqrt{\frac{x^2-4x+3}{x^2-1}} \to$ ~~...~~ $x^2-1\neq 0 \to (x+1)(x-1)\neq 0 \to x\neq\pm 1 \to$ ~~...~~

$D_f=\mathbb{R}-\{\pm 1\}$

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الف) $y=\log(x^2-3x) \to x^2-3x>0 \to x(x-3)>0$ $\quad \frac{0 \quad 3}{+ \; 0 \; - \; 0 \; +} \to$ ~~...~~

$D_f=(-\infty,0)\cup(3,+\infty)$

~~...~~

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ب) $y=\log_{|x|-2}(16-x^2) \to 16-x^2>0 \to (4-x)(4+x)>0$ $\quad \frac{-4 \quad 4}{- \; 0 \; + \; 0 \; -}$

$\to |x|-2>0 \to |x|>2 \to x>2$ یا $x<-2$

$\to |x|-2\neq 1 \to |x|\neq 3 \to x\neq \pm 3$

$D_f=(-4,-2)\cup(2,4)-\{\pm 3\}$

$-4 \quad -2 \quad 2 \quad 4$

ج) $y=\log_{10-x}\frac{x^2-7x+12}{x-3} \to \frac{x^2-7x+12}{x-3}>0 \to \frac{(x-3)(x-4)}{x-3}>0$ $\quad \frac{3^{*} \quad 4}{- \; \| \; - \; 0 \; +}$

$\to 10-x>0 \to 10>x$

$\to 10-x\neq 1 \to 9\neq x$

$D_f=(4,10)-\{9\}$

✓

د) $y=\sqrt{x^2-11x+28}+\log_x \sqrt{36-x^2}$

$\to x^2-11x+28\geq 0 \to (x-4)(x-7)\geq 0$ — ۴ ۷ : $+\ 0\ -\ 0\ +$

$\to \sqrt{36-x^2}>0 \to 36-x^2>0 \to (6-x)(6+x)>0$ — $-6$ ۶ : $-\ 0\ +\ 0\ -$

$\to x>0 \,,\, x\neq 1$

$\to D_f=(0,4]-\{1\}$

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$y=\frac{x^2-x}{x^2+x}$ آپارتمانی

| | | -۱ | | ۰ | | ۱ | |
|---|---|---|---|---|---|---|---|
| $x^2-x$ | + | | + | 0 | − | 0 | + |
| $x^2+x$ | + | 0 | − | 0 | + | | + |
| $y$ | + | ∄ | − | ∄ | − | 0 | + |

سریع: -۱ ۰ ۱ : $+\ ∄\ -\ ∄\ -\ 0\ +$

$y=\frac{x(x-1)}{x(x+1)}$

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$y=\sqrt{\frac{x-f(x)}{f(x)}}$ $\quad \frac{x-f(x)}{f(x)}\geq 0$

$f(x)\neq 0$

$\to x-f(x)=0 \to x=f(x) \to x=3, -1$

$\to f(x)=0 \to x=2, -2$

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-۲ -۱ ۲ ۳ : $-\ ∄\ +\ 0\ -\ ∄\ +\ 0\ -$

$\to D_f=(-\infty ,-2)\cup[-1,2)\cup[3,+\infty)$

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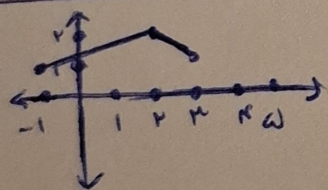
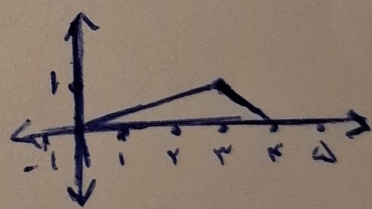
الف) $f(x)+1$



ب) $f(x-1)$



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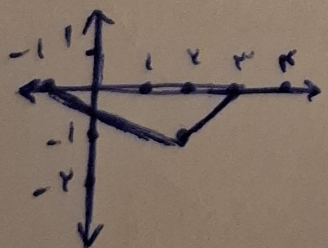
ج) $-f(x)$



د) $f(x+1)+2$