هلیا ترکلیان

۱۸٫۵

۱)

الف) $\dfrac{x+4}{4x^3-20x^2+31x-15} \Rightarrow 4x^3-20x^2+31x-15 \neq 0$

$$\begin{array}{l|l} 4x^3-20x^2+31x-15 & x-1 \\ \underline{-4x^3+4x^2} & \overline{4x^2-16x+15} \\ -16x^2+31x-15 & \\ \underline{+16x^2-16x} & \\ 15x-15 & \\ \underline{-15x+15} & \\ 0 & \end{array}$$

(روش ضربی) $x^2-16x+60$

$(x-10)(x-6)$

$x=\frac{10}{4}, \frac{6}{4} \Rightarrow$

$x=\frac{5}{2}, \frac{3}{2}$

$4x^3-20x^2+31x-15$

$(x-1)(x-\frac{5}{2})(x-\frac{3}{2})$

$D_f = \mathbb{R} - \{1, \frac{5}{2}, \frac{3}{2}\}$

ب) $\dfrac{x+3}{3x^3-x^2-8x-4} \Rightarrow 3x^3-x^2-8x-4 \neq 0$

$$\begin{array}{l|l} 3x^3-x^2-8x-4 & x+1 \\ \underline{-3x^3-3x^2} & \overline{3x^2-4x-4} \\ -4x^2-8x-4 & \\ \underline{+4x^2+4x} & \\ -4x-4 & \\ \underline{+4x+4} & \\ 0 & \end{array}$$

$\xrightarrow{\text{روش}} x^2-4x-12$

$(x-6)(x+2)$

$x=\frac{6}{3}, \frac{-2}{3}$

$\boxed{x=2, \frac{-2}{3}}$

$(x+1)(x-2)(x+\frac{2}{3})$

$D_f = \mathbb{R} - \{-1, 2, -\frac{2}{3}\}$

۲)

الف) $y=\dfrac{x+1}{x-\sqrt{3-2x}}$

$x-\sqrt{3-2x} \neq 0 \rightarrow$ برای اینکه بفهمیم کی صفر

$x-\sqrt{3-2x} \neq 0 \rightarrow$ بشه ریشه رو پیدا میکنیم

$(\sqrt{3-2x})^2 \neq (x)^2$

$3-2x \neq x^2$

$x^2-3+2x \neq 0$

$(x-1)(x+3) \neq 0$ I

$x \neq 1, \boxed{-3} \rightarrow$ این ریشه رو اگر بذاریم صفر نمی‌کنه

$3-2x \geq 0$ II

$2x \leq 3$

$x \leq \frac{3}{2}$

$D_f = (-\infty, \frac{3}{2}] - \{1\}$

ب) $y=\dfrac{x+2}{x-\sqrt{3x-2}}$

$\rightarrow x-\sqrt{3x-2} \neq 0 \rightarrow x-\sqrt{3x-2} \neq 0 \rightarrow (\sqrt{3x-2})^2 \neq (x)^2$

$3x-2 \neq x^2 \rightarrow x^2-3x+2 \neq 0 \rightarrow (x-1)(x-2) \neq 0$

$x \neq 1, 2$

$\rightarrow 3x-2 \geq 0 \rightarrow 3x \geq 2 \rightarrow x \geq \frac{2}{3}$

$D_f = [\frac{2}{3}, +\infty) - \{1, 2\}$

۳

الف) $y = \dfrac{\sin x}{2\cos x - 1}$, $2\cos x - 1 \neq 0 \rightarrow 2\cos x \neq 1 \rightarrow \cos x \neq \dfrac{1}{2}$

$D_f = \mathbb{R} - \{2k\pi + \frac{\pi}{3}, 2k\pi + \frac{5\pi}{3}\}$

ب) $y = \dfrac{\cos x + 1}{2\sin x + 1}$ $\quad 2\sin x + 1 \neq 0 \rightarrow 2\sin x \neq -1 \rightarrow \sin x \neq -\dfrac{1}{2}$

$D_f = \mathbb{R} - \{2k\pi + \frac{7\pi}{6}, 2k\pi + \frac{11\pi}{6}\}$

ج) $\dfrac{\tan x + 2}{\cot x - 1}$

$\rightarrow \tan \rightarrow \dfrac{\sin}{\cos} \rightarrow \cos \neq 0$

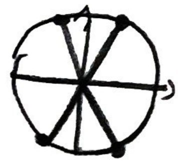
$\rightarrow \cot x \rightarrow \dfrac{\cos}{\sin} \rightarrow \sin \neq 0$

$\rightarrow \cot x - 1 \neq 0 \rightarrow \cot x \neq 1$ $\quad D_f = \mathbb{R} - \{\frac{k\pi}{2}, k\pi + \frac{\pi}{4}\}$

د) $y = \dfrac{\sin x + 1}{4\sin^2 x - 3}$, $4\sin^2 x - 3 \neq 0 \rightarrow 4\sin^2 x \neq 3 \rightarrow \sin^2 x \neq \dfrac{3}{4}$

$\sin x \neq \pm \dfrac{\sqrt{3}}{2}$

$D_f = \mathbb{R} - \{k\pi + \frac{\pi}{3}, k\pi + \frac{2\pi}{3}\}$



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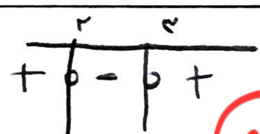
۴

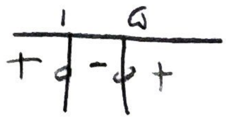
الف) $x^2 - 5x + 6 > 0 \rightarrow (x-2)(x-3) > 0$



$D_f = (-\infty, 2) \cup (3, +\infty)$

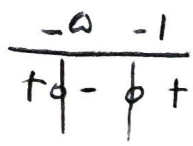
2

ب) $x^2 - 6x + 5 < 0 \rightarrow (x-1)(x-5) < 0$



$D_f = (1, 5)$

ج) $x^2 + 6x + 5 \geq 0 \rightarrow (x+1)(x+5) \geq 0$



$D_f = (-\infty, -5] \cup [-1, +\infty)$

د) $x^2 - 7x + 6 \leq 0 \rightarrow (x-1)(x-6) \leq 0$

$D_f = [1, 6]$

الف) $\dfrac{x^2-3x+2}{x-5}<0 \Rightarrow \dfrac{(x-1)(x-2)}{x-5}<0$

$\begin{array}{ccccccc} & 1 & & 2 & & 5 & \\ - & \circ & + & \circ & - & \circ & + \end{array}$

$D_f=(-\infty,1)\cup(2,5)$

۲

ب) $\dfrac{x^2-1}{x^2-6x+5}>0 \rightarrow \dfrac{(x-1)(x+1)}{(x-1)(x-5)}\geq 0$

$\begin{array}{ccccccc} & -1 & & \overset{*}{1} & & 5 & \\ + & \bullet & - & \circ & - & \circ & + \end{array}$

$D_f=(-\infty,-1]\cup(5,+\infty)$

۵

الف) $y=\sqrt{\dfrac{x^2-4x+3}{x^2-1}} \rightarrow \dfrac{x^2-4x+3}{x^2-1}\geq 0 \rightarrow \dfrac{(x-1)(x-3)}{x^2-1}\geq 0$

۲

$\begin{array}{ccccc} & \overset{*}{1} & & 3 & \\ - & \circ & - & \bullet & + \end{array}$ $D_f=[3,+\infty)$

ب) $y=\sqrt[3]{\dfrac{x^2-4x+3}{x^2-1}} \rightarrow x^2-1\neq 0 \rightarrow x^2\neq 1 \rightarrow x=\pm 1$

$D_f=R-\{\pm 1\}$

۶

الف) $y=\log(x^2-3x) \rightarrow x^2-3x>0 \rightarrow x(x-3)>0$

۳

$\begin{array}{ccccc} & 0 & & 3 & \\ + & \circ & - & \circ & + \end{array}$ $D_f=(-\infty,0)\cup(3,+\infty)$

ب) $y=\log_{|x|-2}(16-x^2)$ $\begin{cases} 16-x^2>0 \rightarrow x^2<16 \rightarrow -4<x<4 \\ |x|-2>0 \rightarrow |x|>2 \rightarrow x>2,\ x<-2 \\ |x|-2\neq 1 \rightarrow |x|\neq 3 \rightarrow x\neq \pm 3 \end{cases}$

$D_f=(-4,-2)\cup(2,4)-\{\pm 3\}$

ج) $y=\log_{10-x}\dfrac{x^2-7x+12}{x-3}$ $\begin{cases} \dfrac{(x-3)(x-4)}{x-3}>0 \quad \begin{array}{ccccc} & \overset{*}{3} & & 4 & \\ - & \circ & - & \circ & + \end{array} \\ 10-x>0 \rightarrow x<10 \\ 10-x\neq 1 \rightarrow x\neq 9 \end{cases}$

$D_f=(4,10)-\{9\}$

۷

د) $y=\sqrt{x^2-11x+28}+\log_x \sqrt{36-x^2}$

① $x^2-11x+28\geq 0 \to (x-4)(x-7)\geq 0$ $\quad \begin{array}{c} 4 \quad 7 \\ + \; 0 \; - \; 0 \; + \end{array}$

② $x>0$, $x\neq 1$

③ $\sqrt{36-x^2}>0 \longrightarrow 36-x^2>0 \to x^2<36 \to -6<x<6$

④ $36-x^2\geq 0 \longrightarrow x^2\leq 36 \to -6\leq x\leq 6$

$D_f=(0,4]-\{1\}$

8 $y=\dfrac{x^2-x}{x^2+x} \to \dfrac{x(x-1)}{x(x+1)}$ ۲

	-1		0		1	
+		−	*	−	0	+

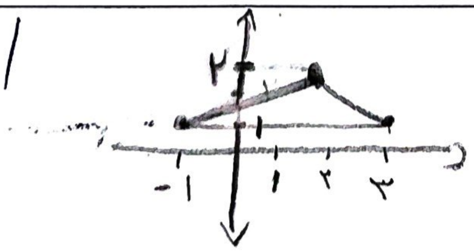
		-1		0		1	
x^2-x	+		+	0	−	0	+
x^2+x	+	0	−	0	+		+
y	+		−		−	0	+

9 $y=\sqrt{\dfrac{x-f(x)}{f(x)}} \to f(x)\neq 0 \longrightarrow x\neq 2$ و -2

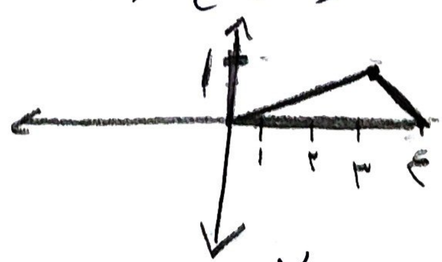
$\dfrac{x-f(x)}{f(x)} \to$ ~~$-2 \; -1 \; 2 \; 3$: $- \; | \; - \; 0 \; + \; | \; + \; 0 \; -$~~ ۰/۵

$D_f=$ ~~$[-1,3]-\{2\}$~~

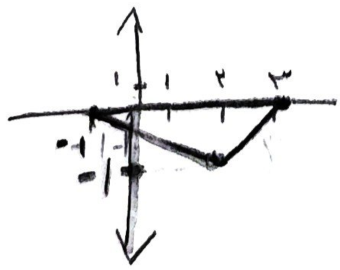
10 الف) $f(x)+1$



ب) $f(x-1)$ ۲



ج) $-f(x)$



د) $f(x+1)+2$