زهرا سادات حسینی

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①

الف) $y = \dfrac{x+4}{4x^3-20x^2+31x-15} =$

جمع ضرایب = ۰ پس بر $x+1$ بخش پذیر

$4x^3-20x^2+31x-15 = (x-1)(2x-5)(2x-3)$

ریشه‌های مخرج = 1 , $\frac{5}{2}$, $\frac{3}{2}$

$$\begin{array}{l|l} 4x^3-20x^2+31x-15 & x+1 \\ \cancel{-4x^3}+4x^2 & 4x^2-16x+15 \\ \hline \cancel{-16x^2}+31x-15 & \\ \cancel{+16x^2}-16x & \\ \hline 15x-15 & \\ -15x+15 & \\ \hline 0 & \end{array}$$

$D_F = \mathbb{R} - \{1, \frac{3}{2}, \frac{5}{2}\}$

ب) $y = \dfrac{x+3}{3x^3-x^2-8x-4} =$

دامنه = $\mathbb{R}$ - ریشه‌های مخرج

$$\begin{array}{l|l} \cancel{3x^3}-x^2-8x-4 & x+1 \\ \cancel{-3x^3}-3x^2 & 3x^2-4x-4 \\ \hline \cancel{-4x^2}-8x-4 & \\ \cancel{+4x^2}+4x & \\ \hline -4x-4 & \\ +4x+4 & \\ \hline 0 & \end{array}$$

روش ضربی

$x^2-4x-12$

$(x-6)(x+2)$

-2 , $\frac{2}{3}$

$D_F = \mathbb{R} - \{+2, -\frac{2}{3}, -1\}$

②

الف) $y = \dfrac{x+1}{x-\sqrt{3-2x}} =$

$x-\sqrt{3-2x} \neq 0 \rightarrow x \neq \sqrt{3-2x}$

$x^2 \neq 3-2x \rightarrow x^2+2x-3 \neq 0 \rightarrow$

$(x-1)(x+3) \neq 0$

ریشه‌ها $\leftarrow$ ① ⓷-3 پاسخ ریشه زائد که صفر نمی‌کند

$D_F = (-\infty, \frac{3}{2}] - \{1\}$

$3-2x \geq 0 \rightarrow 3 \geq 2x \rightarrow \frac{3}{2} \geq x$

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ب) $y=\frac{x+2}{x-\sqrt{3x-2}}=$ $\qquad x-\sqrt{3x-2}\neq 0 \to$

$x\neq\sqrt{3x-2} \to$

$x^2\neq 3x-2 \to x^2-3x+2\neq 0$

$(x-1)(x-2)\neq 0$

$\downarrow \quad \downarrow$

$1 \quad 2$

$3x-2\geq 0$

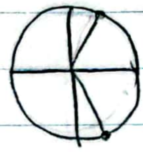
$3x\geq 2$

$x\geq \frac{2}{3}$

$D_f=[\frac{2}{3},+\infty)-\{1,2\}$

③

الف) $y=\frac{\sin x}{2\cos x-1} \to 2\cos x-1\neq 0 \to 2\cos x\neq 1 \to \cos x\neq \frac{1}{2}$



$D_f=\mathbb{R}-\{2\pi k\pm\frac{\pi}{3}\}$

ب) $y=\frac{\cos x+1}{2\sin x+1} \to 2\sin x+1\neq 0 \to \sin x\neq -\frac{1}{2}$

$D_f=\mathbb{R}-\{2k\pi-\frac{\pi}{6},\ 2k\pi+\frac{7\pi}{6}\}$

ج) $y=\frac{\tan x+2}{\cot x-1}=\frac{\frac{\sin x}{\cos x}+2}{\frac{\cos x}{\sin x}-1}=$

$D_f=\mathbb{R}-\{\frac{k\pi}{2},\ k\pi+\frac{\pi}{4}\}$

① $\cos x\neq 0$

② $\sin x\neq 0$

③ $\cot x-1\neq 0 \to \cot x\neq 1$

د) $y=\frac{\sin x+1}{4\sin^2 x-3} \to 4\sin^2 x-3\neq 0 \to \sin^2 x\neq\frac{3}{4} \to \sin x\neq\pm\frac{\sqrt{3}}{2}$

$D_f=\mathbb{R}-\{k\pi+\frac{\pi}{3},\ k\pi+\frac{2\pi}{3}\}$

④

الف) $x^2-5x+6\geq 0 \to (x-2)(x-3)\geq 0$

$\downarrow \quad \downarrow$

$2 \quad 3$

	2		3	
+	0	−	0	+

$D_f=(-\infty,2]\cup[3,+\infty)$

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ب) $x^2-6x+5<0 \rightarrow (x-1)(x-5)<0$

ریشه‌ها: 1 , 5

$$\begin{array}{ccccc} & 1 & & 5 & \\ + & \vert & - & \vert & + \end{array}$$

$D_f=(1,5)$

ج) $x^2+6x+5\geq 0 \rightarrow (x+1)(x+5)\geq 0$

-1 , -5

$$\begin{array}{ccccc} & -5 & & -1 & \\ + & \vert & - & \vert & + \end{array}$$

$D_f=(-\infty,-5]\cup[-1,+\infty)$

د) $x^2-7x+6\leq 0 \rightarrow (x-1)(x-6)\leq 0$

1 , 6

$$\begin{array}{ccccc} & 1 & & 6 & \\ + & \vert & - & \vert & + \end{array}$$

$D_f=[1,6]$

5

الف) $\dfrac{x^2-3x+2}{x-5}<0 \rightarrow \dfrac{(x-1)(x-2)}{x-5}<0 \rightarrow$

1 , 2 , 5

$$\begin{array}{ccccccc} & 1 & & 2 & & 5 & \\ - & \vert & + & \vert & - & \vert & + \end{array}$$

$D_f=(-\infty,1)\cup(2,5)$

ب) $\dfrac{x^2-1}{x^2-6x+5}\geq 0 \rightarrow \dfrac{(x-1)(x+1)}{(x-1)(x-5)}\geq 0$

1 , -1 , 1 , 5

$$\begin{array}{ccccccc} & -1 & & *1 & & 5 & \\ + & \vert & - & \vert & - & \vert & + \end{array}$$

$D_f=(-\infty,-1]\cup(5,+\infty)$

6

الف) $y=\sqrt{\dfrac{x^2-4x+3}{x^3-1}}=\sqrt{\dfrac{(x-1)(x-3)}{(x-1)(x^2+x+1)}}$

1 , 3 , 1

ریشه ندارد

$$\begin{array}{ccccc} & *1 & & 3 & \\ - & \vert & - & \vert & + \end{array}$$

$D_f=[3,+\infty)$

ب) $y=\sqrt[3]{\dfrac{x^2-4x+3}{x^2-1}}=$

$(x-1)(x+1)\neq 0$

1 , -1

$D_f=\mathbb{R}-\{1,-1\}$

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7)

الف) $y=\log(x^2-3x) \to x^2-3x>0 \to x(x-3)>0$

$\downarrow 0 \quad \downarrow 3$

	0		3	
+	0	−	0	+

$D_F=(-\infty,0]\cup[3,+\infty)$

ب) $y=\log_{|x|-2}^{16-x^2} = 0<16-x^2 \to x^2<16 \to -4<x<4$

$0<|x|-2 \to 2<|x| \to 2<x$

$x<-2$

$|x|-2\neq 1 \to |x|\neq 3 \to x\neq \pm 3$

$D_F=(-4,-2)\cup(2,4)-\{\pm 3\}$

ج) $y=\log_{10-x}^{\frac{x^2-7x+12}{x-3}}$ = ① $\frac{x^2-7x+12}{x-3}>0 \to \frac{(x-3)(x-4)}{(x-3)}>0$

	*3		4	
−	0	−	0	+

② $10-x>0 \to 10>x$

③ $10-x\neq 1 \to 9\neq x$

$D_F=(4,10)-\{9\}$

د) $y=\sqrt{x^2-11x+28}+\log_x^{\sqrt{36-x^2}} = (x-7)(x-4)\geq 0$

$\downarrow 7 \quad \downarrow 4$

	4		7	
+	0	−	0	+

$\sqrt{36-x^2}>0 \to \sqrt{(6-x)(6+x)}>0$

	−6		6	
−	0	+	0	−

$x>0$

$x\neq 1$

$D_F=(0,4]-1$

8)

$y=\frac{x^2-x}{x^2+x}=\frac{x(x-1)}{x(x+1)}$

	−1		*0		1	
+	0	−	0	−	0	+

	−1	0	1	
x^2-x	+	+	−	+
x^2+x	+	−	+	+
	+	−	−	+

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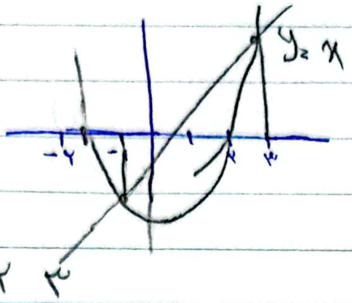
⑨

$y=\sqrt{\dfrac{x-f(x)}{f(x)}}$

ریشه‌ها با توجه به شکل

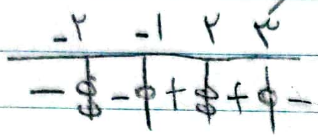
$-1 , 3$



$D_f=[-1 , 3]-\{2\}$

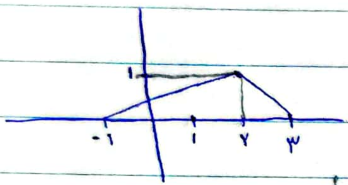
$f(x)\neq 0$

$x\neq 2 , -2$

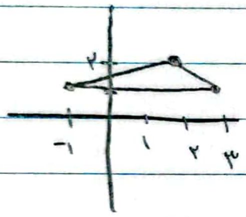


با توجه به شکل، مخرج در نقاط ۲ و -۲ صفر می‌شود.

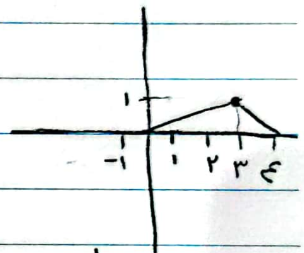
⑩



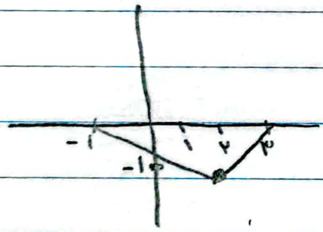
الف) $f(x)+1$



ب) $f(x-1)$



ج) $-f(x)$



د) $f(x+1)+2$