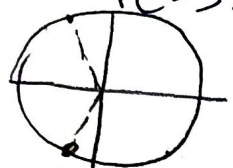


الف) $y = \frac{\sin x - 1}{\cos x + 1} \Rightarrow \cos x + 1 \neq 0 \rightarrow \cos x \neq -1$

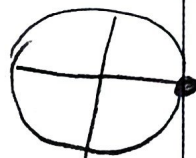


$D_f = \mathbb{R} - \left\{ 2k\pi + \frac{3\pi}{2}, 2k\pi + \frac{5\pi}{2} \right\}$

$\Rightarrow y = \frac{\sin x + 1}{\cos x - 1}$

$\Rightarrow \cos x - 1 \neq 0 \rightarrow \cos x \neq 1$

$D_f = \mathbb{R} - \{ 2k\pi \}$



الف) $y = \frac{\sin x + 1}{\tan x + 1}$

$\Rightarrow \begin{cases} \tan x \rightarrow \frac{\sin}{\cos} \rightarrow \cos \neq 0 \\ \tan x + 1 \neq 0 \rightarrow \tan \neq -1 \end{cases}$

$D_f = \mathbb{R} - \left\{ k\pi + \frac{3\pi}{2}, k\pi + \frac{\pi}{2} \right\}$



$\Rightarrow y = \frac{\cos x + 1}{\cot x - 1}$

$\Rightarrow \begin{cases} \cot x \rightarrow \frac{\cos}{\sin} \rightarrow \sin \neq 0 \\ \cot x - 1 \neq 0 \rightarrow \cot \neq 1 \end{cases}$

$D_f = \mathbb{R} - \left\{ k\pi, k\pi + \frac{\pi}{2} \right\}$



الف) $\sin y = x^2 - 1 \Rightarrow -1 \leq x^2 - 1 \leq 1 \rightarrow 0 \leq x^2 \leq 2 \rightarrow 1 \leq x \leq \sqrt{2}$

$\sqrt{1} \leq x \leq \sqrt{2} \rightarrow D_f = [1, \sqrt{2}] \cup [-\sqrt{2}, -1]$

$\downarrow -1 \geq x \geq -\sqrt{2}$

ب) $\arccos(\sqrt{x} - 1)$

$\begin{cases} x \geq 0 \\ 1 \leq \sqrt{x} - 1 \leq 1 \end{cases} \rightarrow 1 \leq \sqrt{x} \leq 2 \rightarrow 1 \leq x \leq 4$

$1 \leq x \leq 4$

$D_f = [1, 4]$

$$a) \cos y = |x| - r \rightarrow -1 \leq |x| - r \leq 1 \rightarrow r \leq |x| \leq r+1$$

$$\begin{cases} r \leq x \leq r+1 \\ -r \geq x \geq -r-1 \end{cases} \rightarrow D_f = [r, r+1] \cup [-r-1, -r]$$

$$b) y = \arcsin(x^r + r x + 1) \Rightarrow -1 \leq x^r + r x + 1 \leq 1$$

$$\begin{cases} x^r + r x + 1 \leq 1 \rightarrow x^r + r x \leq 0 \Rightarrow x(x+r) \leq 0 \quad I \\ -1 \leq x^r + r x + 1 \rightarrow 0 \leq x^r + r x + 2 \rightarrow 0 \leq (x+1)(x+r) \quad II \end{cases}$$

$$\begin{array}{c|c|c} -r & 0 & \\ \hline + & 0 & - & 0 & + \end{array} \quad I \quad \begin{array}{c|c|c} -r-1 & -1 & \\ \hline + & 0 & - & 0 & + \end{array} \quad II \quad \text{I \& II} \Rightarrow D_f = [-r, -r] \cup [-1, 0]$$

$$c) y = \log_r(x^r - r) \begin{cases} x^r - r > 0 \\ r > 0 \\ r \neq 1 \end{cases} \Rightarrow x^r > r \rightarrow x > r \text{ or } x < -r$$

$$D_f = (-\infty, -r) \cup (r, +\infty)$$

$$d) y = \log_r(r - |x|) \begin{cases} r - |x| > 0 \\ r > 0 \\ r \neq 1 \end{cases} \Rightarrow |x| < r \rightarrow -r < x < r$$

$$D_f = (-r, r)$$

$$e) y = \log \frac{a-x}{x-r} \begin{cases} a-x > 0 \rightarrow x < a \\ x-r > 0 \rightarrow x > r \\ x-r \neq 1 \rightarrow x \neq r+1 \end{cases} D_f = (r, a) - \{r+1\}$$

$$f) y = \log \frac{x^r - 1}{x + r} \begin{cases} x^r - 1 > 0 \rightarrow x^r > 1 \rightarrow \boxed{x > 1 \text{ or } x < -1} \\ x + r > 0 \rightarrow x > -r \\ x + r \neq 0 \rightarrow x \neq -r \end{cases}$$

$$D_f = (-r, -1) \cup (1, +\infty) - \{-r\}$$

$$g) y = \log \frac{x^r - r x + r}{x - r} \rightarrow \frac{x^r - r x + r}{x - r} > 0 \quad \frac{(x-1)(x-r)}{x-r} > 0$$

$$\begin{array}{c|c|c} 1 & r & \\ \hline - & 0 & + & 0 & + \end{array} \quad D_f = (1, +\infty) - \{r\}$$

$$\begin{aligned}
 \text{ب) } y &= \log \frac{x+r}{x-a} \\
 D_f &= (-a-r) \cup (r, +\infty) \cdot \left\{ \begin{array}{l} \frac{x+r}{x-a} > 0 \rightarrow \frac{-r}{-a-r} + \\ x+a > 0 \rightarrow x > -a \\ x+a \neq 1 \rightarrow x \neq -r \end{array} \right.
 \end{aligned}$$

$$\begin{aligned}
 \text{ج) } y &= \sqrt{r - \log_r x} \Rightarrow r - \log_r x \geq 0 \rightarrow \log_r x \leq r \\
 \left\{ \begin{array}{l} x-r > 0 \rightarrow x > r \\ r > 0 \\ r \neq 1 \end{array} \right. & \quad x-r \leq 1 \rightarrow \boxed{x \leq 10} \\
 D_f &= (r, 10]
 \end{aligned}$$

$$\begin{aligned}
 \text{د) } y &= \log(r \log_r x - 1) \\
 D_f &= (\sqrt{r}, +\infty) \quad \left\{ \begin{array}{l} r \log_r x - 1 > 0 \rightarrow \log_r x > \frac{1}{r} \\ x > 0 \end{array} \right. \\
 & \quad \downarrow \\
 & \quad x > \sqrt{r}
 \end{aligned}$$

$$\text{هـ) } y = \frac{r}{e^n + 1} \rightarrow e^n + 1 \neq 0 \rightarrow e^n \neq -1 \rightarrow D = \mathbb{R}$$

$$\text{و) } y = \frac{r}{e^n - 1} \rightarrow e^n - 1 \neq 0 \rightarrow e^n \neq 1 \rightarrow x \neq 0 \rightarrow D_f = \mathbb{R} - \{0\}$$

$$\begin{aligned}
 \text{ز) } y &= \frac{r}{e^n - r} \rightarrow e^n - r \neq 0 \rightarrow e^n \neq r \rightarrow x \neq \log_e r \\
 D_f &= \mathbb{R} - \{\log_e r\}
 \end{aligned}$$

$$\begin{aligned}
 \text{ح) } y &= \frac{r}{e^n - r} \rightarrow e^n - r \neq 0 \rightarrow e^n \neq r \rightarrow x \neq \log_e r \\
 D_f &= \mathbb{R} - \{\log_e r\}
 \end{aligned}$$

$$\text{ط) } y = (en+1)! \Rightarrow en+1 \in \mathbb{W} \rightarrow n = \frac{W-1}{e}$$

$$D_f = \left\{ n \mid n = \frac{k-1}{e}, k \in \mathbb{W} \right\}$$

$$\text{ي) } y = \frac{r^n - r}{r^n - a} \Rightarrow \frac{r^n - r}{r^n - a} \in \mathbb{W} \rightarrow r^n - r = r^n W - a W \rightarrow r^n - r^n W = r - a W$$

$$D_f = \left\{ n \mid n = \frac{r - aW}{r - rW}, W \in \mathbb{W} \right\} \quad \leftarrow \quad n = \frac{r - aW}{r - rW}$$