

الف) $y = \frac{\sin x - 2}{2\cos x + 1} \Rightarrow 2\cos x + 1 \neq 0$
 $2\cos x \neq -1$
 $\cos x \neq -\frac{1}{2} \rightarrow x \neq 120^\circ, 240^\circ$

$DF = R - \left\{ 2k\pi + \frac{4\pi}{3}, 2k\pi + \frac{2\pi}{3} \right\}$

ب) $y = \frac{\sin x + 3}{\cos x - 1} \Rightarrow \cos x - 1 \neq 0$
 $\cos x \neq 1 \Rightarrow x \neq 0^\circ (360^\circ)$

$DF = R - \{2k\pi\}$

الف) $y = \frac{2\sin x + 1}{\tan x + 1} \Rightarrow \tan x + 1 \neq 0$
 $\frac{\sin x}{\cos x} + 1 \neq 0 \Rightarrow \frac{\sin x}{\cos x} \neq -1 \Rightarrow x \neq 135^\circ, 315^\circ$



$\cos x \neq 0 \Rightarrow x \neq 90^\circ, 270^\circ$

$DF = R - \left\{ k\pi + \frac{\pi}{4}, k\pi + \frac{3\pi}{4} \right\}$

ب) $y = \frac{\cos x + 1}{\cot x - 1} \Rightarrow \cot x - 1 \neq 0$
 $\cot x \neq 1 \Rightarrow \frac{\cos x}{\sin x} \neq 1 \Rightarrow x \neq 45^\circ, 225^\circ$

$\sin x \neq 0 \Rightarrow x \neq 0^\circ, 180^\circ$



$DF = R - \left\{ k\pi, k\pi + \frac{\pi}{4} \right\}$

الف) $\sin y = x^2 - 2 \Rightarrow -1 \leq \sin y \leq 1 \Rightarrow -1 \leq x^2 - 2 \leq 1$

$DF: ① \cap ② \cap ③ \Rightarrow [-\sqrt{3}, \sqrt{3}], [1, +\infty), (-\infty, -1]$

$DF: [-\sqrt{3}, -1] \cup [1, \sqrt{3}]$

$DF: [-\sqrt{3}, \sqrt{3}]$

ب) $y = \arccos \sqrt{x - 3} \Rightarrow x - 3 \geq 0$
 $[3, +\infty): \sqrt{x - 3} \leq 1 \Rightarrow x - 3 \leq 1 \Rightarrow x \leq 4$

$DF: ① \cap ② \Rightarrow [3, 4]$

$DF: (-\infty, 4]$

الف) $\cos y = |x| - 3 \Rightarrow -1 \leq |x| - 3 \leq 1$

$DF: ① \cap ② \cap ③ \Rightarrow [-2, 2] \cap [3, +\infty) \cap (-\infty, -2]$
 $\Rightarrow DF = [-2, -2] \cup [2, 2]$

$|x| - 3 \leq 1 \Rightarrow |x| \leq 4 \Rightarrow -4 \leq x \leq 4$

$|x| - 3 \geq -1 \Rightarrow |x| \geq 2 \Rightarrow x \leq -2 \text{ or } x \geq 2$

$DF: [-4, -2] \cup [2, 4]$

ب) $y = \arcsin (x^2 + 3x + 1) \Rightarrow -1 \leq x^2 + 3x + 1 \leq 1$

$-1 \leq x^2 + 3x + 1 \Rightarrow x^2 + 3x + 2 \geq 0 \Rightarrow (x+1)(x+2) \geq 0 \Rightarrow x \leq -2 \text{ or } x \geq -1$

$x^2 + 3x + 1 \leq 1 \Rightarrow x^2 + 3x \leq 0 \Rightarrow x(x+3) \leq 0 \Rightarrow -3 \leq x \leq 0$

$DF: [-3, -2] \cup [-1, 0]$

الف) $y = \log_{\frac{1}{3}} (x^2 - 4) \Rightarrow x^2 - 4 > 0 \Rightarrow (x-2)(x+2) > 0 \Rightarrow x < -2 \text{ or } x > 2$

$DF: (-\infty, -2) \cup (2, +\infty)$

ب) $y = \log_2 \frac{1-x}{1+x} \Rightarrow \frac{1-x}{1+x} > 0$

$1-x > 0 \Rightarrow x < 1$
 $1+x > 0 \Rightarrow x > -1$

$DF: (-1, 1)$

$\frac{1-x}{1+x} > 0 \Rightarrow \frac{1-x}{1+x} > -2 \Rightarrow 1-x > -2(1+x) \Rightarrow 1-x > -2-2x \Rightarrow x > -3$

$x = (-3, 1)$

الف) $y = \log_{\alpha} \frac{\alpha^2 - 1}{\alpha - 3} \Rightarrow \frac{\alpha^2 - 1}{\alpha - 3} > 0 \Rightarrow \alpha > 3 \quad (1)$
 $\alpha - 3 > 0 \Rightarrow \alpha > 3 \quad (2)$
 $\alpha - 3 \neq 1 \Rightarrow \alpha \neq 4 \quad (3)$
 $DF: (3, 4) \cup (4, +\infty)$

ب) $y = \log_{\alpha} \frac{\alpha^2 - 1}{\alpha + 3} \Rightarrow \frac{\alpha^2 - 1}{\alpha + 3} > 0 \Rightarrow (\alpha - 1)(\alpha + 1) > 0 \Rightarrow \alpha < -1 \text{ or } \alpha > 1$
 $\alpha + 3 > 0 \Rightarrow \alpha > -3 \Rightarrow (-3, +\infty) \quad (2)$
 $\alpha + 3 \neq 1 \Rightarrow \alpha \neq -2 \quad (3)$
 $DF: (-3, -2) \cup (-2, 1) \cup (1, +\infty)$

الف) $y = \log_{\alpha} \frac{\alpha^2 - 5\alpha + 6}{\alpha - 2} \Rightarrow \frac{\alpha^2 - 5\alpha + 6}{\alpha - 2} > 0 \Rightarrow \frac{(\alpha - 1)(\alpha - 3)}{\alpha - 2} > 0$
 $10 > 0$
 $10 \neq 1$
 $DF: (1, 2) \cup (3, +\infty)$
 $\alpha = (1, 2) \cup (3, +\infty)$

ب) $y = \log_{\alpha} \frac{\alpha + 3}{\alpha - 2} \Rightarrow \frac{\alpha + 3}{\alpha - 2} > 0 \Rightarrow \alpha < -3 \text{ or } \alpha > 2$
 $\alpha + 3 > 0 \Rightarrow \alpha > -3 \quad (2)$
 $\alpha + 3 \neq 1 \Rightarrow \alpha \neq -4 \quad (3)$
 $DF: (-3, -4) \cup (2, +\infty)$

الف) $y = \sqrt{3 - \log_2(\alpha - 2)} \Rightarrow 3 - \log_2(\alpha - 2) \geq 0 \Rightarrow \log_2(\alpha - 2) \leq 3 \Rightarrow \alpha - 2 \leq 8 \Rightarrow \alpha \leq 10$
 $\alpha - 2 > 0 \Rightarrow \alpha > 2 \quad (2)$
 $DF: (2, 10]$

ب) $y = \log_2(2 \log_2 \alpha - 1) \Rightarrow 2 \log_2 \alpha - 1 > 0 \Rightarrow \log_2 \alpha > \frac{1}{2} \Rightarrow \alpha > 2^{\frac{1}{2}} = \sqrt{2}$
 $DF: (\sqrt{2}, +\infty)$

الف) $\frac{\alpha}{\alpha + 1} \Rightarrow \alpha \neq -1 \Rightarrow DF: \mathbb{R}$

ب) $\frac{\alpha}{\alpha - 1} \Rightarrow \alpha \neq 1 \Rightarrow \alpha \neq 0 \Rightarrow DF: \mathbb{R} - \{0\}$

ج) $y = \frac{\alpha}{\alpha - 2} \Rightarrow \alpha \neq 2 \Rightarrow \alpha \neq \log_2 2 \Rightarrow DF: \mathbb{R} - \{\log_2 2\}$

د) $\frac{\alpha}{\alpha - 3} \Rightarrow \alpha \neq 3 \Rightarrow \alpha \neq \log_2 3 \Rightarrow DF: \mathbb{R} - \{\log_2 3\}$

الف) $y = (2\alpha + 1)! \Rightarrow 2\alpha + 1 \in \mathbb{N} \Rightarrow 2\alpha \in \mathbb{N} - 1 \Rightarrow \alpha \in \frac{\mathbb{N} - 1}{2}$

$DF: \{\alpha \mid \alpha = \frac{k-1}{2}, k \in \mathbb{N}\}$

ب) $y = (\frac{\alpha - 2}{\alpha - 5})! \Rightarrow \frac{\alpha - 2}{\alpha - 5} \in \mathbb{N} \Rightarrow \alpha - 2 \in \mathbb{N}(\alpha - 5) \Rightarrow \alpha - 2 \in \mathbb{N} - 5\mathbb{N}$

مثلاً: $\frac{\alpha - 2}{\alpha - 5} \in \mathbb{N} \Rightarrow \alpha = \frac{5\mathbb{N} - 2}{\mathbb{N} - 3}$

$\alpha \in \frac{5\mathbb{N} - 2}{\mathbb{N} - 3} \Rightarrow \alpha \in \frac{5\mathbb{N} - 2}{\mathbb{N} - 3}$

$DF: \{\alpha \mid \alpha = \frac{5k-2}{k-3}, k \in \mathbb{N}\}$