نام و نام خانوادگی ..... پاسخنامه تشریحی تکلیف شماره ۳ ..... کلاس .....

۱

الف) $y^3 - x = 2x^2 \Rightarrow y_1^3 = 2x^2 + x \rightarrow y_1^3 = y_2^4 \rightarrow y_1 = y_2$ تابع هست، تعریف ریاضی

$y_2^3 = 2x^2 + x$

ب) $|y| + x^2 = x + 3 \rightarrow x = 0 \Rightarrow |y| = 3 < \begin{matrix} y = 3 \\ y = -3 \end{matrix}$ تابع نیست: مثال نقض

ج) $\underbrace{\sqrt{x-4}}_{\text{مثبت}} + \underbrace{|y-2|}_{\text{مثبت}} = 0 \Rightarrow \sqrt{x-4} = 0 \Rightarrow x - 4 = 0 \Rightarrow |x = 4|$ ، $|y-2| = 0 \Rightarrow |y = 2|$ تابع هست: تک نقطه

د) $x = \cos y \Rightarrow x = 0 \rightarrow \cos y = 0 \Rightarrow y = \{90^\circ, -90^\circ\} \Rightarrow \cos 90^\circ, \cos 270^\circ$ تابع نیست: مثال نقض

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الف) $y = \frac{x+4}{x^2-4x} \Rightarrow x^2 - 4x = 0 \Rightarrow x(x-4) = 0 \Rightarrow x = 0 \text{ یا } x = 4 \Rightarrow D_f = \mathbb{R} - \{4, 0\}$

ب) $y = \frac{x+4}{x^2-7x+12} \Rightarrow x^2 - 7x + 12 = 0 \Rightarrow \Delta = b^2 - 4ac \Rightarrow (-7)^2 - (4 \times 1 \times 12) = -23 \Rightarrow \Delta < 0$ ریشه ندارد $D_f = \mathbb{R}$

ج) $y = \frac{x+4}{x^2+x+4} \Rightarrow x^2 + x + 4 = 0 \Rightarrow \Delta = b^2 - 4ac = 1^2 - (4 \times 1 \times 4) = -15 \Rightarrow \Delta < 0$ ریشه ندارد $: D_f = \mathbb{R}$

د) $y = \frac{x+4}{x^4 - 4x^2} \Rightarrow x^4 - 4x^2 = 0 \Rightarrow x^2(x^2 - 4) \Rightarrow x^2(x+2)(x-2) \Rightarrow x = 0 \text{ یا } x = 2 \text{ یا } x = -2$

$D_f = \mathbb{R} - \{0, 2, -2\}$

۳

الف) $y = \sqrt{4-x} + \sqrt{x-2} \Rightarrow \begin{matrix} \sqrt{4-x} \geq 0 \Rightarrow 4 - x \geq 0 \Rightarrow 4 \geq x \\ \sqrt{x-2} \geq 0 \Rightarrow x - 2 \geq 0 \Rightarrow x \geq 2 \end{matrix} \left\{ \begin{matrix} (-\infty, 4] \\ [2, +\infty) \end{matrix} \right\} \cap \Rightarrow D_f = [2, 4]$

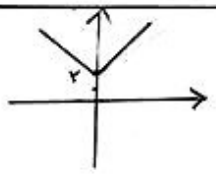
ب) $y = \frac{3x+1}{x^2+1} \Rightarrow \underbrace{x^2}_{\text{مثبت}} + \underbrace{1}_{\text{مثبت}} = 0 \Rightarrow$ ریشه ندارد $\Rightarrow D_f = \mathbb{R}$

ج) $y = \frac{\sqrt{x-2}}{x-4} \Rightarrow \sqrt{x-2} \geq 0 \Rightarrow x - 2 \geq 0 \Rightarrow x \geq 2 \text{ ، } x - 4 \neq 0 \Rightarrow x \neq 4 \Rightarrow D_f = [2, 4) \cup (4, +\infty)$

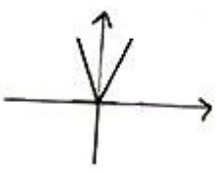
د) $y = \frac{2x+3}{|x| + x^2} \Rightarrow \underbrace{|x|}_{\text{مثبت}} + \underbrace{x^2}_{\text{مثبت}} = 0 \Rightarrow x = 0 \Rightarrow D_f = \mathbb{R} - \{0\}$

۴

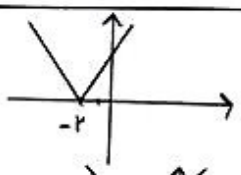
الف) $y = |x| + 2$



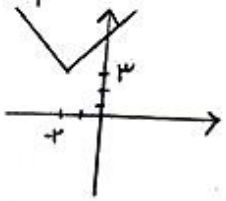
ب) $y = 2|x|$



ج) $y = |x+2|$

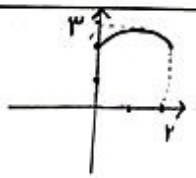


د) $y = |x+2| + 3$

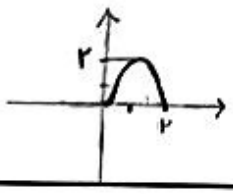


۵

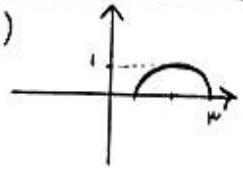
الف) $F(x) + 2$



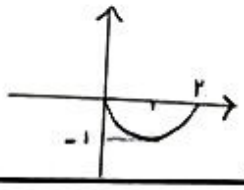
ب) $2F(x)$



ج) $F(x-1)$

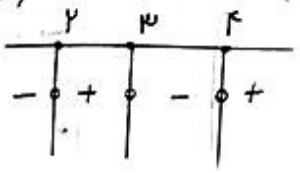


د) $-F(x)$

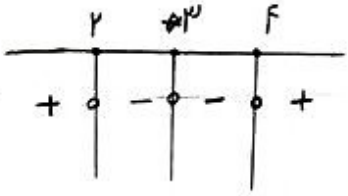


۶

الف) $y=(x-2)(x-3)(x-4) \rightarrow \underline{x-2=0 \Rightarrow |x=2|} \quad \underline{x-3=0 \Rightarrow |x=3|} \quad \underline{x-4=0 \Rightarrow |x=4|}$

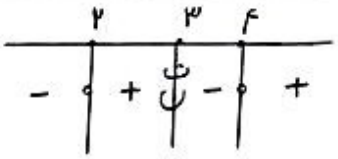


ب) $y=(x-2)(x-3)^2(x-4) \rightarrow \underline{x-2=0 \Rightarrow |x=2|} \quad \underline{x-3=0 \Rightarrow |x=3|} \quad \underline{x-4=0 \Rightarrow |x=4|}$

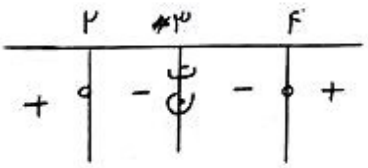


۷

الف) $y=\dfrac{(x-2)(x-4)}{x-3} \Rightarrow \underline{x-2=0 \Rightarrow |x=2|} \quad \underline{x-4=0 \Rightarrow |x=4|} \quad \underline{x-3=0 \Rightarrow |x=3|}$



ب) $y=\dfrac{(x-2)(x-4)}{(x-3)^2} \quad \underline{x-2=0 \Rightarrow |x=2|} \quad \underline{x-4=0 \Rightarrow |x=4|} \quad \underline{x-3=0 \Rightarrow |x=3|}$



۸

الف) $y=\dfrac{x^3-3x+2}{x-3}$ ← $1-3+2=0$ یعنی $x-1$ عامل است

$x^3-3x+2 \;|\; x-1$

$-x^3+x^2$

x^2-3x+2

$-x^2+x$

$-2x+2$

$+2x-2$

0

$x^2+x-2 \rightarrow (x+2)(x-1)$

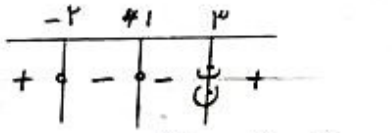
$y=\dfrac{(x-1)(x-1)(x+2)}{(x-3)} \rightarrow -2$

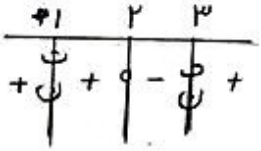
$1 \leftarrow (x-1)$

$1 \rightarrow (x-1)$

$3 \leftarrow (x-3)$

$\underline{x-1=0 \Rightarrow |x=1|} \quad \underline{x+2=0 \Rightarrow |x=-2|} \quad \underline{x-3=0 \Rightarrow |x=3|}$



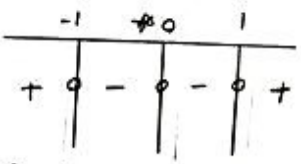


ب) $y=\dfrac{x^2-3x+2}{x^2-4x+3}=\dfrac{(x-1)(x-2)}{(x-1)(x-3)}=0 \quad \underline{x-1=0 \Rightarrow |x=1|} \quad \underline{x-2=0 \Rightarrow |x=2|} \quad \underline{x-3=0 \Rightarrow |x=3|}$

۹

الف) $y=\dfrac{x^4-x^2}{x^2+x+2}=\dfrac{x^2(x^2-1)}{x^2+x+2}=\dfrac{x^2(x+1)(x-1)}{x^2+x+2} \rightarrow \underline{x=0} \quad \underline{x+1=0 \Rightarrow |x=-1|} \quad \underline{x-1=0 \Rightarrow |x=1|}$

$\Delta=b^2-4ac=1^2-(4\times2\times1)=-7 \Rightarrow \Delta<0$ ریشه ندارد



ب) $y=\dfrac{x^2+3x+4}{x^2+2x+3} \rightarrow \Delta=b^2-4ac=3^2-(4\times1\times4)=9-16=-7 \rightarrow \Delta<0$ ریشه ندارد

$\Delta=b^2-4ac=2^2-(4\times3\times1)=4-12=-8 \rightarrow \Delta<0$ ریشه ندارد

$x\in\mathbb{R} \Rightarrow$ به ازای همه مقادیر مثبت است ←

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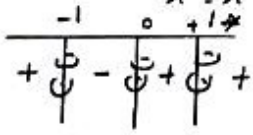
الف) $y=\sqrt{\dfrac{x^3-1}{x^3-x}}=\sqrt{\dfrac{(x-1)(x^2+x+1)}{x(x^2-1)}}\cdot\sqrt{\dfrac{(x-1)(x^2+x+1)}{(x)(x+1)(x-1)}} \rightarrow \Delta=b^2-4ac=1^2-(4\times1\times1)=-3<0$ ریشه ندارد

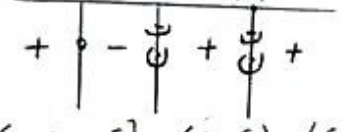
$\underline{x-1=0 \Rightarrow |x=1|} \quad \underline{|x=0|} \quad \underline{x+1=0 \Rightarrow |x=-1|}$



$\Rightarrow Df.\ (-\infty,-1)\cup(0,+1)\cup(+1,+\infty)=(-\infty,-1)\cup(0,+\infty)$

ب) $y=\sqrt{\dfrac{x^2-16}{x^2-5x+4}}\cdot\sqrt{\dfrac{(x+4)(x-4)}{(x-1)(x-4)}}$

$\underline{x+4=0 \Rightarrow |x=-4|} \quad \underline{x-1=0 \Rightarrow |x=1|} \quad \underline{x-4=0 \Rightarrow |x=4|}$



$Df.\ (-\infty,-4]\cup(1,4)\cup(4,+\infty)$