

الف) $\left. \begin{aligned} y_1^3 - x &= 2x^2 \\ y_2^3 - x &= 2x^2 \end{aligned} \right\} y_1^3 = y_2^3 \rightarrow y_1 = y_2 \rightarrow y^3 - x = 2x^2$ تابع (تغییراتی)

ب) $x=0 \quad |y|=3 \quad y=\pm 3 \rightarrow |y|+x^2 = x+3$ تابع نیست (مثال نقض)

ج) $\sqrt{x-4} + |y-2| = 0 \quad \sqrt{x-4}=0 \rightarrow x=4 \quad |y-2|=0 \rightarrow y=2$ تابع است (نقطه ۴, ۲)
 رسم بردار

د) $x = \cos y \quad x=0 \rightarrow y = k\pi + \frac{\pi}{2} \rightarrow$ تابع نیست (مثال نقض)

الف) $y = \frac{x+2}{x^2-4x} \rightarrow$ تابع منفرجه $\rightarrow x^2-4x=0 \quad x(x-4)=0 \quad x=0 \quad x-4=0 \quad x=4 \quad D_f = R - \{0, 4\}$

ب) $y = \frac{x+2}{x^2-4x+4} \rightarrow$ تابع منفرجه $\rightarrow x = \frac{1 \pm \sqrt{1-4+4}}{2} \rightarrow D_f = R$

ج) $y = \frac{x+2}{x^2+x+2} \rightarrow$ تابع منفرجه $\rightarrow x = \frac{-1 \pm \sqrt{1-4+8}}{2} \rightarrow D_f = R$

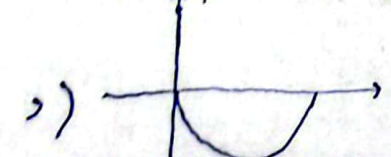
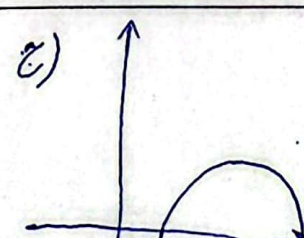
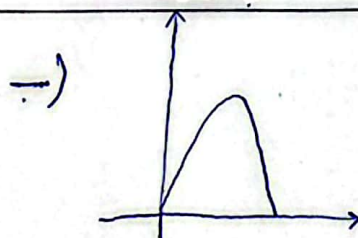
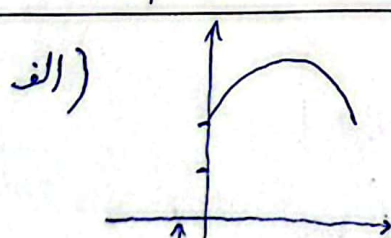
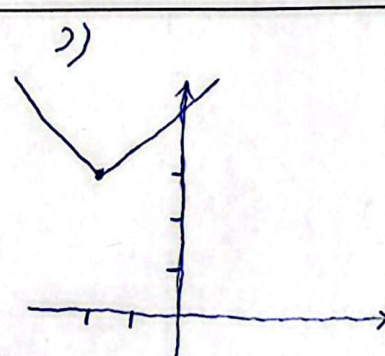
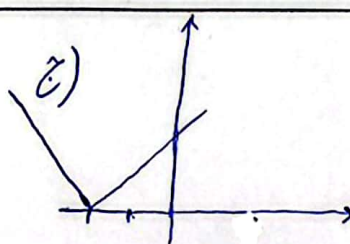
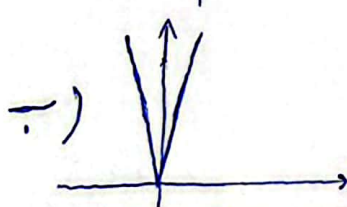
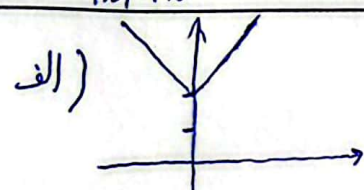
د) $y = \frac{x+2}{x^2-4x^2} \rightarrow$ تابع منفرجه $\rightarrow x^2-4x^2 = x^2(x^2-4) = x^2(x-2)(x+2) = 0 \quad x=0 \quad x=2 \quad x=-2 \quad D_f = R - \{0, 2, -2\}$

الف) $\sqrt{4-x} + \sqrt{x-2} \rightarrow \begin{aligned} 4-x &\geq 0 & x-2 &\geq 0 \\ x &\leq 4 & x &\geq 2 \end{aligned} \rightarrow D_f = [2, 4]$

ب) $\frac{x^2+1}{x^2+1} \rightarrow$ تابع همواره ۱ است $\rightarrow D_f = R$

ج) $y = \frac{\sqrt{x+2}}{x-4} \rightarrow x-2 \geq 0 \quad x \geq 2, \quad x-4 \neq 0 \quad x \neq 4 \rightarrow D_f = [2, \infty) - \{4\}$

د) $\frac{x^2+3}{|x|+x^2} \rightarrow |x|=0, \quad x^2=0 \rightarrow x=0 \rightarrow D_f = R - \{0\}$



$$\text{الف) } y = (x-2)(x-3)(x-4)$$

$$\begin{array}{r} 2 \quad 3 \quad 4 \\ - \quad \phi + \quad \phi - \quad \phi + \end{array}$$

$$\text{ب) } y = (x-2)(x-3)^2(x-4)$$

$$\begin{array}{r} 2 \quad 3 \quad 4 \\ + \quad \phi - \quad \phi - \quad \phi + \end{array}$$

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$$\text{الف) } y = \frac{(x-2)(x-4)}{x-3}$$

$$\begin{array}{r} 2 \quad 3 \quad 4 \\ - \quad \phi + \quad \phi - \quad \phi + \end{array}$$

$$\text{ب) } y = \frac{(x-2)(x-4)}{(x-3)^2}$$

$$\begin{array}{r} 2 \quad 3 \quad 4 \\ + \quad \phi - \quad \phi - \quad \phi + \end{array}$$

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$$\text{الف) } y = \frac{x^3 - 3x + 2}{x-3} = \frac{(x-1)^2(x+2)}{x-3}$$

$$\begin{array}{r} -2 \quad 1 \quad 3 \\ + \quad \phi - \quad \phi - \quad \phi + \end{array}$$

$$\begin{array}{r} x^3 - 3x + 2 \mid x-1 \\ -x^2 + x^2 \\ \hline x^2 - 3x + 2 \\ -x^2 + x \\ \hline -2x + 2 \\ 2x - 2 \\ \hline 0 \end{array} \quad \begin{array}{r} x-1 \\ x^2 + x - 2 \\ \hline (x-1)(x+2) \end{array}$$

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$$\text{ب) } y = \frac{x^3 - 3x + 2}{x^2 - 2x + 3} = \frac{(x-1)(x-2)}{(x-1)(x-3)}$$

$$\begin{array}{r} 1 \quad 2 \quad 3 \\ + \quad \phi + \quad \phi - \quad \phi + \end{array}$$

$$\text{الف) } y = \frac{x^4 - x^2}{x^2 + x + 2} = \frac{x^2(x^2 - 1)}{x^2 + x + 2} = \frac{x^2(x-1)(x+1)}{x^2 + x + 2}$$

$$\Delta = 1 - 1 = -1$$

$$\begin{array}{r} -1 \quad 0 \quad 1 \\ + \quad \phi - \quad \phi - \quad \phi + \end{array}$$

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$$\text{ب) } y = \frac{x^2 + 2x + 4}{x^2 + 2x + 3} \rightarrow \Delta = 9 - 16 = -7$$

$$\Delta = 4 - 12 = -8$$

$$\text{الف) } y = \sqrt{\frac{x^2 - 1}{x^2 - x}} = \sqrt{\frac{(x-1)(x+1)}{x(x^2 - 1)}} = \sqrt{\frac{(x-1)(x+1)}{x(x-1)(x+1)}} \quad D_f = (-\infty, -1) \cup (0, 1) \cup (1, \infty)$$

$$\begin{array}{r} -1 \quad 0 \quad 1 \\ - \quad \phi - \quad \phi - \quad \phi + \end{array} \rightarrow \begin{array}{r} -1 \quad 0 \quad 1 \\ - \quad \phi + \quad \phi - \quad \phi + \end{array} \rightarrow \begin{array}{r} -1 \quad 0 \quad 1 \\ + \quad \phi - \quad \phi + \quad \phi + \end{array}$$

$$\text{ب) } \sqrt{\frac{x^2 - 16}{x^2 - 8x + 4}} = \sqrt{\frac{(x-4)(x+4)}{(x-1)(x-4)}}$$

$$\begin{array}{r} (x-4)(x+4) \mid (x-1)(x-4) \\ \hline + \quad \phi - \quad \phi - \quad \phi + \end{array}$$

$$D_f = (-\infty, -4] \cup (1, 4) \cup (4, \infty)$$

$$\begin{array}{r} -4 \quad 1 \quad 4 \\ + \quad \phi - \quad \phi + \quad \phi + \end{array}$$

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