$f(x) = \begin{cases} x^2+2x \; ; & x \geq a \\ ax-4 \; ; & x < a \end{cases}$ پیوسته باشد

$a^2+2a = a^2-4 \Rightarrow 2a=-4$

$a=-2$ ①

$f(x) = \dfrac{x^2+a}{2x-b}$ $\quad g(x) = 2x+b$

$g(2)=3 \Rightarrow 4+b=3 \Rightarrow b=-1$

$f(2)=3 \Rightarrow \dfrac{x^2+a}{2x+1} \Rightarrow \dfrac{4+a}{4+1}=3 \quad \dfrac{4+a}{5}=3$ $\quad a=11$ ②

$f(1) \Rightarrow \dfrac{1+11}{2+1} = \dfrac{12}{3} = 4$

$f(x) = \dfrac{4x+1}{2x^2+ax+b}$ $\quad D_f = \mathbb{R}-\{4,-1\} \Rightarrow$ ریشه مخرج

$S = 3 \Rightarrow a=-6$ $\quad P = -4 \Rightarrow b=-8$ $\quad f(x) = \dfrac{4x+1}{2x^2-6x-8}$ ③

$f(1) = \dfrac{4+1}{2-6-8} \Rightarrow \dfrac{5}{-12}$

$f(x) = \dfrac{x^3-\sqrt{3}}{-4x^2+ax+b}$ $\quad D_f = \mathbb{R}-\{1\} \Rightarrow$ ریشه مضاعف

$\dfrac{-a}{-4} = 2 \Rightarrow a=-8$ $\quad \dfrac{b}{-4}=1 \quad b=-4$ $\quad a+b \Rightarrow -8-4=-12$ ④

$f(x) = \dfrac{2x}{(x-1)(x^2+mx+1)}$ $\quad D_f = \mathbb{R}-\{1\}$

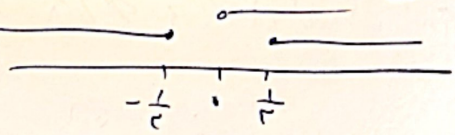
حالت اول: $\Delta<0 \Rightarrow m^2-4<0 \quad (m+2)(m-2)<0 \quad m=(-2,2)$ (1) ⑤

$\Delta=0 \Rightarrow m^2-4=0 \quad m=\pm 2$ $\quad m=\pm 2$ (2)

$(1) \cap (2) = [-2,2]$

$f(x)=\sqrt{\varepsilon-\frac{1}{x^{2}}} \Leftrightarrow \sqrt{\frac{\varepsilon x^{2}-1}{x^{2}}}$ ⇔ $\varepsilon x^{2}-1 \geq 0$ $\varepsilon x^{2} \geq 1$ $x^{2} \geq \frac{1}{\varepsilon}$

$x \geq \frac{1}{2}$ $x \leq -\frac{1}{2}$ $x^{2} \geq 0 \Rightarrow x \geq 0$

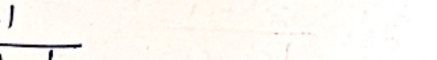


④

$\Leftrightarrow D_f = [\frac{1}{2}, +\infty)$

$f(x)=\sqrt{mx^{2}+2xm+1}$ $mx^{2}+2xm+1 \geq 0$ $(2m)^{2}-\varepsilon(m)(1) \leq 0$

$\varepsilon m^{2}-\varepsilon m \leq 0$ $\varepsilon m(m-1) \leq 0$



⑤

$m \in [0,1]$

$f(x)=\begin{cases} \frac{\varepsilon x^{2}-1}{2x-1} ; x \neq a \\ 2x+k ; x=\frac{1}{2} \end{cases}$ $g(x)=2x+1$

$g(\frac{1}{2})=2(\frac{1}{2})+k \Rightarrow \underbrace{\varepsilon(\frac{1}{2})+1}_{2} = \underbrace{\varepsilon(\frac{1}{2})}_{2}+\underbrace{k}_{0}$

⑦

$2x-1=0$ $2x=1$ $x=\frac{1}{2} \Leftrightarrow a=\frac{1}{2}$ $a+k=\frac{1}{2}+0=\frac{1}{2}$ ✗

$f(x)=\begin{cases} \frac{9x^{2}-\varepsilon}{2x+2} ; x \neq \frac{2}{3} \\ 2ax+1 ; x=-\frac{2}{3} \end{cases}$ $g(x)=2a+b$

$x=-\frac{2}{3} \Leftrightarrow$ $\varepsilon(-\frac{2}{3})a+2=\varepsilon(-\frac{2}{3})+b$

$-2a+2=-2+b$ ①

$x=1 \Rightarrow \frac{9-\varepsilon}{2+2}=2+b \Rightarrow 1=2+b$ $b=-1$ ②

①,② ⇒ $-2a+2=-2-2 \Leftrightarrow -2a=-4$ $a=2$

$a-b=2+1=3$ ✗

⑨

$g(1)=f(2) \Rightarrow 2+2=2a^{2}+2$

$2a^{2}+2a-\varepsilon=0 \Rightarrow a^{2}+a-2=0$

$(a+2)(a-1)=0$

-2 $+1$

$a=+1,-2$ ✗

⑩