مهرانا فراغتی - دهم دختر C - تکلیف شماره ۲۸

1

$$f(x)=\begin{cases} x^2+2x & ; x\geqslant a \\ ax-4 & ; x\leqslant a \end{cases} \qquad f(a)=a^2+2a \qquad f(a)=a^2-4$$

$$a^2+2a=a^2-4 \rightsquigarrow 2a=-4$$

$$\rightsquigarrow a=\frac{-4}{2}=-2 \rightsquigarrow \boxed{a=-2}$$

2

$$f(x)=\frac{x^2+a}{2x-b} \quad , \quad g(x)=2x+b$$ متقاطع اند $(2,3)$ »

$$\rightsquigarrow f(2)=\frac{2^2+a}{2\times 2-b}=\frac{4+a}{4-b}=3 \rightsquigarrow 4+a=12-3b$$

$$a=8-3b$$

$$g(2)=2\times 2+b=4+b=3 \rightsquigarrow b=3-4=-1 \rightsquigarrow b=-1$$

$$a=8-3b=8-3(-1)=8+3=11 \rightsquigarrow a=11$$

$$f(x)=\frac{x^2+11}{2x+1} \rightsquigarrow f(1)=\frac{1^2+11}{2\times 1+1}=\frac{1+11}{2+1}=\frac{12}{3}=4 \rightsquigarrow \boxed{f(1)=4}$$

3

$$f(x)=\frac{4x+1}{2x^2+ax+b} \qquad D_f=\mathbb{R}-\{-1,4\}$$

$\{-1,4\} \rightsquigarrow$ ریشه‌های مخرج هستند.

$$2(-1)^2+a(-1)+b=0 \rightsquigarrow 2-a+b=0$$

$$2(4)^2+a(4)+b=0 \rightsquigarrow 32+4a+b=0$$

$$2-a+\cancel{b}=32+4a+\cancel{b} \rightsquigarrow -30=5a \rightsquigarrow a=-6$$

$$\rightsquigarrow 2-(-6)+b=0 \rightsquigarrow 8+b=0 \rightsquigarrow b=-8$$

$$f(x)=\frac{4x+1}{2x^2-6x-8} \rightsquigarrow f(1)=\frac{4+1}{2-6-8}=\frac{5}{-12} \rightsquigarrow \boxed{f(1)=-\frac{5}{12}}$$

4

$$f(x)=\frac{x^3-\sqrt{3}}{-4x^2+ax+b} \qquad D_f=\mathbb{R}-\{-1\}$$

$\{-1\} \rightsquigarrow$ ریشه مضاعف ریشه است

$$(x+1)^2=x^2+2x+1 \overset{\times(-4)}{\rightsquigarrow} -4x^2-8x-4$$

$$\rightsquigarrow -4x^2+ax+b=-4x^2-8x-4$$

$$ax+b=-8x-4 \rightsquigarrow a=-8 \quad , \quad b=-4$$

$$\rightsquigarrow a+b=-8-4=-12 \rightsquigarrow \boxed{a+b=-12}$$

5

$$f(x)=\frac{2x}{(x-1)(x^2+mx+1)} \qquad D_f=\mathbb{R}-\{1\}$$

$\rightsquigarrow x^2+mx+1 \rightsquigarrow$ ریشه ندارد

$$b^2-4ac<0 \rightsquigarrow m^2-4\times 1\times 1<0 \rightsquigarrow m^2-4<0$$

$$\rightsquigarrow m^2<4 \rightsquigarrow -2<m<2 \rightsquigarrow m=(-2,2)$$

$\rightsquigarrow x^2+mx+1 \rightsquigarrow$ ریشه مضاعف ۱ دارد

$$(x-1)^2=x^2-2x+1 \rightsquigarrow \cancel{x^2}+mx+\cancel{1}=\cancel{x^2}-2x+\cancel{1}$$

$$\rightsquigarrow mx=-2x \rightsquigarrow m=-2 \rightsquigarrow \boxed{m=[-2,2)}$$

6

$$f(x)=\sqrt{4-\frac{1}{x^2}} \rightsquigarrow 4-\frac{1}{x^2}\geqslant 0 \rightsquigarrow 4\geqslant\frac{1}{x^2}$$

$$\rightsquigarrow \frac{1}{4}\leqslant x^2 \rightsquigarrow -\frac{1}{2}\leqslant x\leqslant\frac{1}{2} \rightsquigarrow \boxed{x=\left[-\frac{1}{2},\frac{1}{2}\right]}$$

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$$f(x)=\sqrt{mx^2+2mx+1} \rightsquigarrow D_f=\mathbb{R}$$

$a>0$

$m>0$

$mx^2+2mx+1 \rightsquigarrow$ ریشه مضاعف دارد $\rightsquigarrow$ $+ \quad 0 \quad +$

$$\rightsquigarrow b^2-4ac=0 \rightsquigarrow (2m)^2-4(m)(1)=0 \rightsquigarrow 4m^2-4m=0$$

$$\rightsquigarrow m^2-m=0 \rightsquigarrow m(m-1)=0 \rightsquigarrow m=\{0,1\}$$

$mx^2+2mx+1 \rightsquigarrow$ ریشه نداشته باشد $\rightsquigarrow$ $+$

$$b^2-4ac<0 \rightsquigarrow (2m)^2-4(m)(1)<0 \rightsquigarrow 4m^2-4m<0$$

$$\rightsquigarrow 4m^2<4m \rightsquigarrow m^2<m \rightsquigarrow m<1$$

$\rightsquigarrow m>0$ $(a>0)$ $\rightsquigarrow$

$$\left.\right\}\ (-\infty,1)\cap(0,+\infty) = (0,1)$$

$$\rightsquigarrow m=\{0,1\}\cap\ 0<m<1 \rightsquigarrow \boxed{\begin{matrix}0\le m\le 1\\ m=[0,1]\end{matrix}}$$

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$$f(x)=\begin{cases}\dfrac{4x^2-1}{2x-1} & ;\ x\neq a\\ 4x+k & ;\ x=\dfrac{1}{2}\end{cases} \hookrightarrow D_f=\mathbb{R} \quad , \quad g(x)=2x+1 \hookrightarrow D_g=\mathbb{R}$$

$$2x-1=0 \rightsquigarrow 2x=1 \rightsquigarrow x=\frac{1}{2} \qquad g\left(\frac{1}{2}\right)=2\times\frac{1}{2}+1=1+1=2$$

$$\rightsquigarrow x\neq\frac{1}{2} \rightsquigarrow a=\frac{1}{2} \qquad 2+k=2 \rightsquigarrow k=0$$

$$f\left(\frac{1}{2}\right)=4\times\frac{1}{2}+k=2+k \qquad \rightsquigarrow a+k=\frac{1}{2}+0=\boxed{\frac{1}{2}}$$

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$$f(x)=\begin{cases}\dfrac{9x^2-4}{3x+2} & ;\ x\neq-\dfrac{2}{3}\\ 3ax+2 & ;\ x=-\dfrac{2}{3}\end{cases} \quad , \quad g(x)=3x+b \downarrow D_g=\mathbb{R}$$

$$\rightsquigarrow D_f=\mathbb{R}$$

$$f(1)=\frac{9-4}{3+2}=\frac{5}{5}=1 \quad \rightsquigarrow 3+b=1 \rightsquigarrow b=-2$$

$$g(1)=3+b$$

$$f\left(-\frac{2}{3}\right)=3\times\left(-\frac{2}{3}\right)\times a+2=-2a+2$$

$$g\left(-\frac{2}{3}\right)=3\left(-\frac{2}{3}\right)+b=-2-2=-4$$

$$\rightsquigarrow -2a+2=-4 \rightsquigarrow -2a=-6 \rightsquigarrow a=3$$

$$a-b=3-(-2)=3+2=5 \rightsquigarrow \boxed{a-b=5}$$

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$$f(x)=\begin{cases}\dfrac{x^2-4}{x-2} & ;\ x\neq 2\\ 2a^2+ax & ;\ x=2\end{cases} \quad , \quad g(x)=x+2$$

$$f(2)=2a^2+2a \rightsquigarrow 2a^2+2a=4$$

$$g(2)=2+2=4 \qquad a^2+a=2$$

$$\rightsquigarrow a^2+a-2=0 \rightsquigarrow 1a^2+1a-2=0$$

$$b^2-4ac=1^2-4\times1(-2)=1+8=9$$

$$\frac{-b\pm\sqrt{\Delta}}{2a}=\frac{-1\pm\sqrt{9}}{2}=\frac{-1\pm3}{2} \rightsquigarrow \frac{-4}{2}=-2 \quad ; \quad \frac{2}{2}=1$$

$$\boxed{a=\{-2,1\}}$$